Principles of Communications

Chapter 3: Analog Modulation
(continued)

Textbook: Ch 3, Ch 4.1-4.4

3.3 Angle Modulation

- Angle modulation is either phase or frequency of the carrier is varied according to the message signal
- The general form of an angle modulated wave is

$$s(t) = A_c \cos\left[2\pi f_c t + \theta(t)\right]$$

where f_c = carrier freq, $\theta(t)$ is the time-varying phase and varied by the message m(t)

- The instantaneous frequency of s(t) is

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

Representation of FM and PM signals

- For phase modulation (PM), we have

$$\theta(t) = k_p m(t)$$

where k_p = phase deviation constant

- For frequency modulation (FM), we have

$$f_i(t) - f_c = k_f m(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

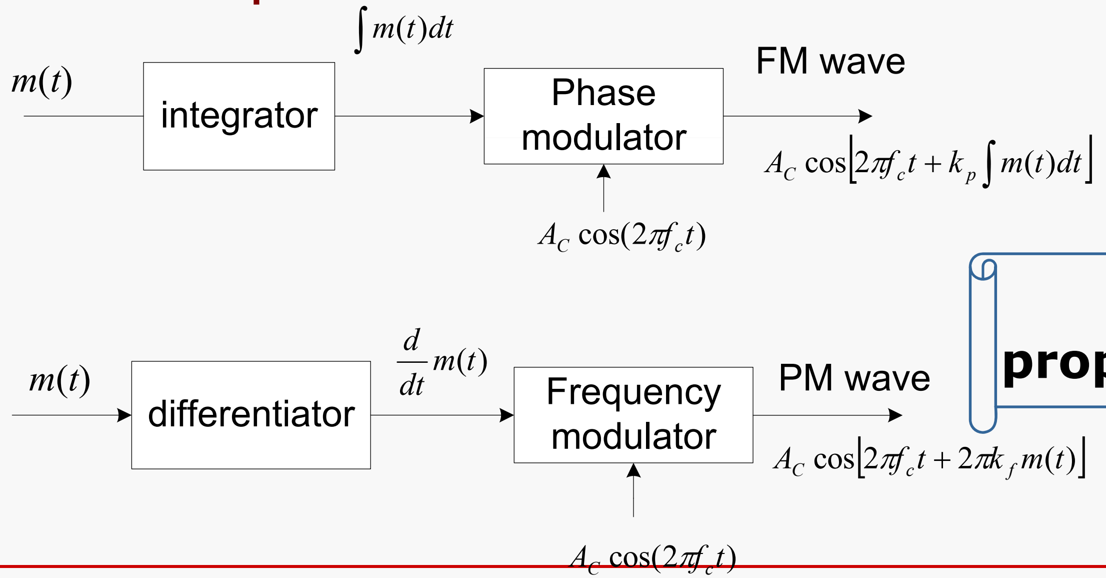
where k_f = frequency deviation constant

- The phase of FM is

$$\theta(t) = 2\pi k_f \int_0^t m(\tau)d\tau$$

Distinguishing Features of PM and FM

- No perfect regularity in spacing of zero crossing
  - Zero crossings refer to the time instants at which a waveform changes between negative and positive values
- Constant envelop, i.e. amplitude of s(t) is constant
- Relationship between PM and FM



Discuss the properties of FM only

Example: Sinusoidal Modulation

Sinusoid modulating wave m(t)

FM wave

$$\frac{d}{dt}m(t)$$

PM wave

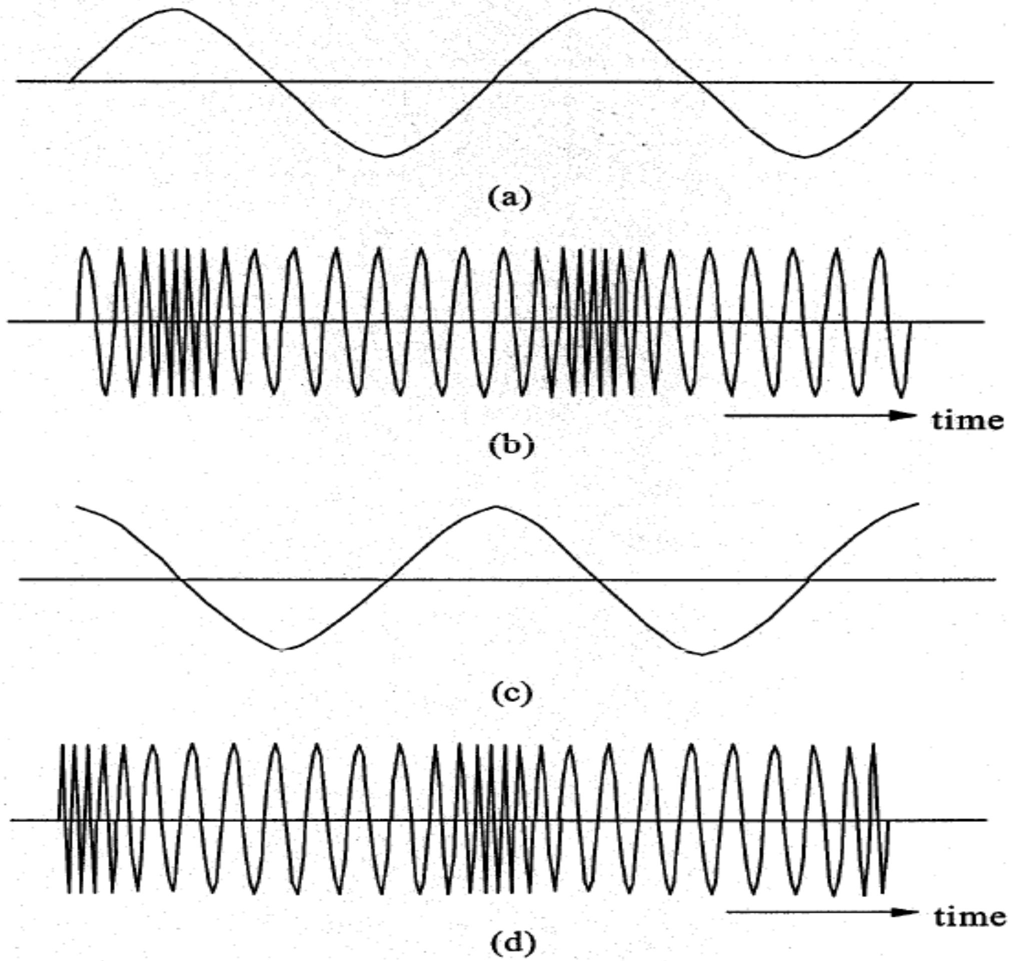


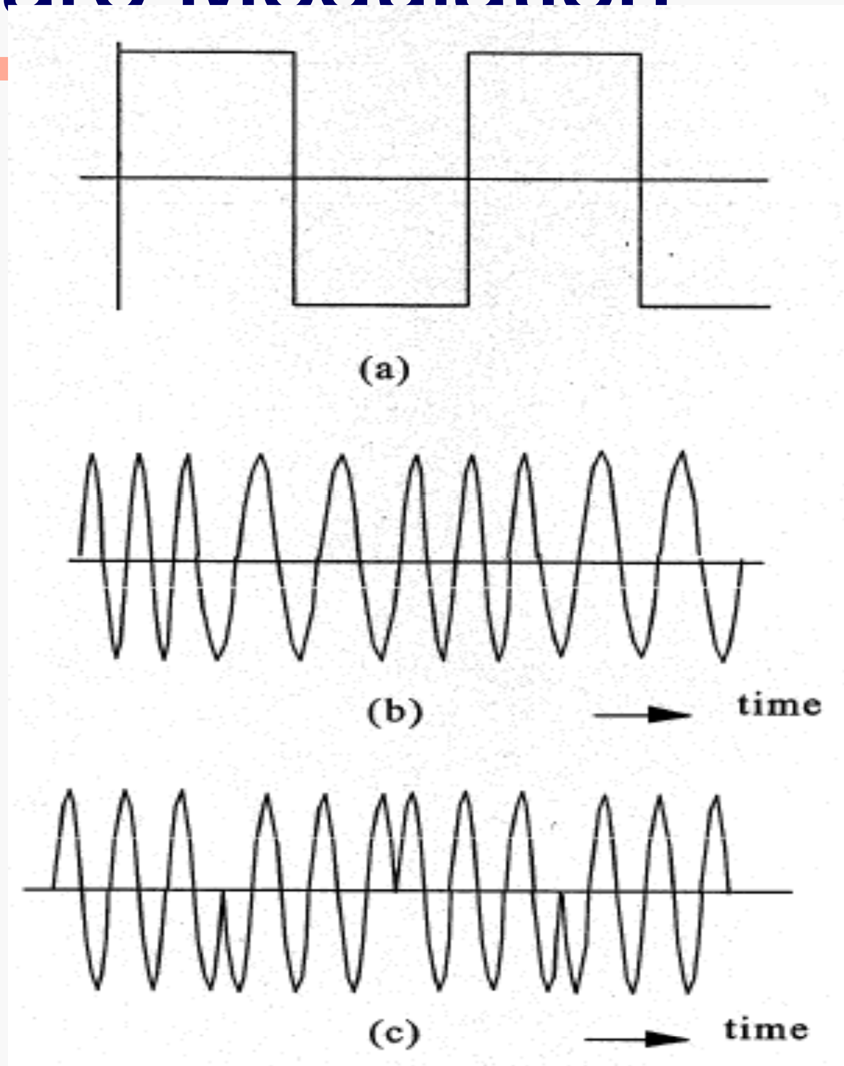
Figure 8.2 - (a) Sinusoidal modulation wave m(t). (b) FM wave. (c) Derivative of *m(t)* with respect to time. (d) PM wave.

Example: Square Modulation

Square modulating wave m(t)

FM wave

PM wave



FM by a Sinusoidal Signal

- Consider a sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

- Instantaneous frequency of resulting FM wave is

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t) = f_c + \Delta f \cos(2\pi f_m t)$$

where $\Delta f = k_f A_m$ is called the frequency deviation, proportional to the amplitude of $m(t)$, and independent of f_m.

- Hence, the carrier phase is

$$\theta(t) = 2\pi \int_0^t \left(f_i(\tau) - f_c \right) d\tau = \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$$= \beta \sin(2\pi f_m t)$$

Where $\beta = \Delta f / f_m$ is called the modulation index

Example

- Problem: a sinusoidal modulating wave of amplitude 5V and frequency 1kHz is applied to a frequency modulator. The frequency sensitivity is 40Hz/V. The carrier frequency is 100kHz. Calculate (a) the frequency deviation, and (b) the modulation index

- Solution:
  - Frequency deviation $\Delta f = k_f A_m = 40 \times 5 = 200 Hz$
  - Modulation index $\beta = \dfrac{\Delta f}{f_m} = \dfrac{200}{1000} = 0.2$

Spectrum Analysis of Sinusoidal FM Wave

- The FM wave for sinusoidal modulation is

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$$
$$= \underbrace{A_c \cos\left[\beta \sin(2\pi f_m t)\right]}_{\text{In-phase component}} \cos(2\pi f_c t) - \underbrace{A_c \sin\left[\beta \sin(2\pi f_m t)\right]}_{\text{Quadrature-phase component}} \sin(2\pi f_c t)$$

In-phase component

$$s_I(t) = A_c \cos\left[\beta \sin(2\pi f_m t)\right]$$

Quadrature-phase component

$$s_Q(t) = A_c \sin\left[\beta \sin(2\pi f_m t)\right]$$

- Hence, the complex envelop of FM wave is

$$\tilde{s}(t) = s_I(t) + j s_Q(t) = A_c e^{j\beta \sin(2\pi f_m t)}$$

- $\tilde{s}(t)$ retains complete information about s(t)

$$s(t) = \mathrm{Re}\left\{A_c e^{j\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]}\right\} = \mathrm{Re}\left[\tilde{s}(t) e^{j2\pi f_c t}\right]$$

- $\tilde{s}(t)$ is periodic, can be expanded in Fourier series as

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

$$\tilde{s}(t) = A_c e^{j\beta \sin(2\pi f_m t)}$$

where

$$c_n = f_m \int_{-1/(2f_m)}^{1/(2f_m)} \tilde{s}(t) e^{-j2\pi n f_m t} dt$$

$$= f_m A_c \int_{-1/(2f_m)}^{1/(2f_m)} e^{j[\beta \sin(2\pi f_m t) - 2\pi n f_m t]} dt$$

- Let $x = 2\pi f_m t$

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp\left[j\left(\beta \sin x - nx\right)\right] dx$$

- n-th order Bessel function of the first kind $J_n(\beta)$ is defined as

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\left[j\left(\beta \sin x - nx\right)\right] dx$$

- Hence,

$$c_n = A_c J_n(\beta)$$

- Substituting c_n into $\tilde{s}(t)$

$c(t) = A_c J_n(\beta)$

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\exp\left(j2\pi n f_m t\right)$$

- Hence, FM wave in time domain can be represented by

$$s(t) = A_c \operatorname{Re}\left\{ \sum_{n=-\infty}^{\infty} J_n(\beta)\exp\left[j2\pi(f_c + nf_m)t \right] \right\}$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos\left[2\pi(f_c + nf_m)t\right]$$

- In frequency-domain, we have

$$S(f) = \frac{A_c}{2}{}_c \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right]$$

- Property 1: Narrowband FM (for small $\beta \leq 0.3$)
  - Approximations

$$J_0(\beta) \approx 1$$
$$J_1(\beta) \approx \beta/2$$
$$J_n(\beta) \approx 0,\ n > 1$$

  - Substituting above into s(t)

$$s(t) \approx A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2}\cos[2\pi(f_c + f_m)t] - \frac{\beta A_c}{2}\cos[2\pi(f_c - f_m)t]$$

? In what ways do a conventinal AM wave and a narrow band FM wave differ from each other

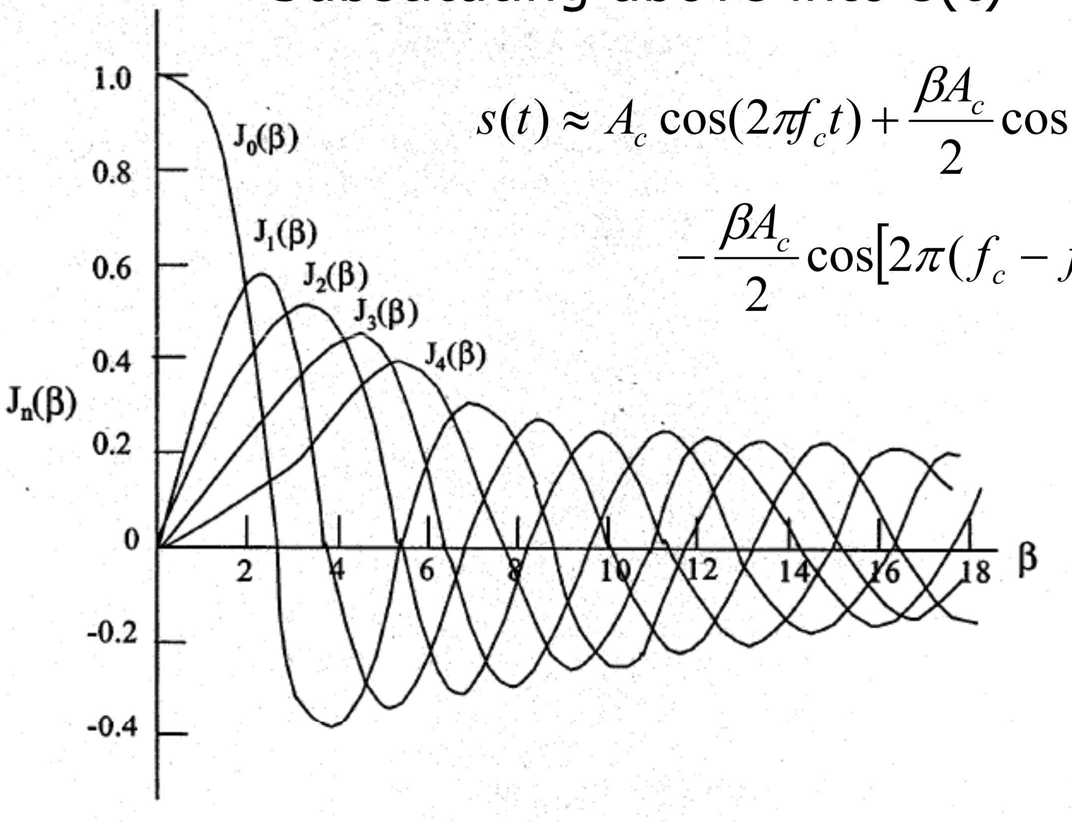


Figure 8.4 - Plots of Bessel functions of the first kind.

$$|J_n(\beta)| \to 0 \text{ as } \beta \to \infty$$

$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases}$$

- Property 2: Wideband FM (for large β>1)
  - In theory, s(t) contains a carrier and an infinite number of side-frequency components, with no approximations made
- Property 3: Constant average power
  - The envelop of FM wave is constant, so the average power is also constant, $P = A_c^2 / 2$
  - The average power is also given by

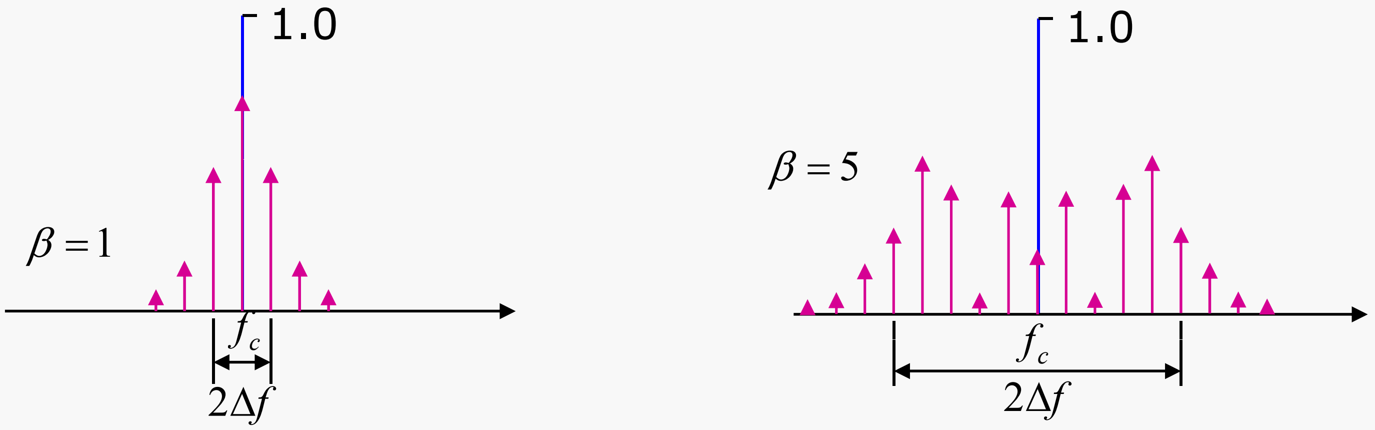
$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)]$$

$$P = \frac{A_c^2}{2} \sum_n J_n^2(\beta) = \frac{A_c^2}{2}$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

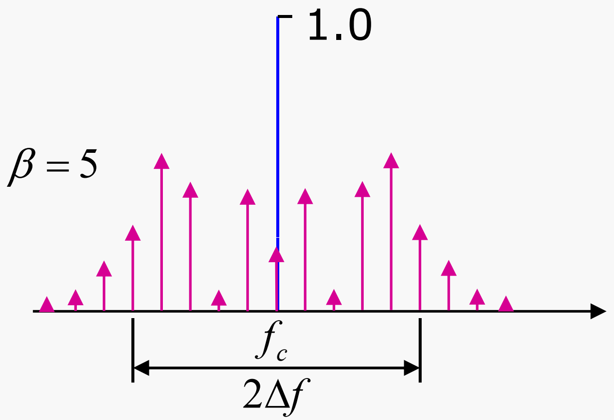
Example

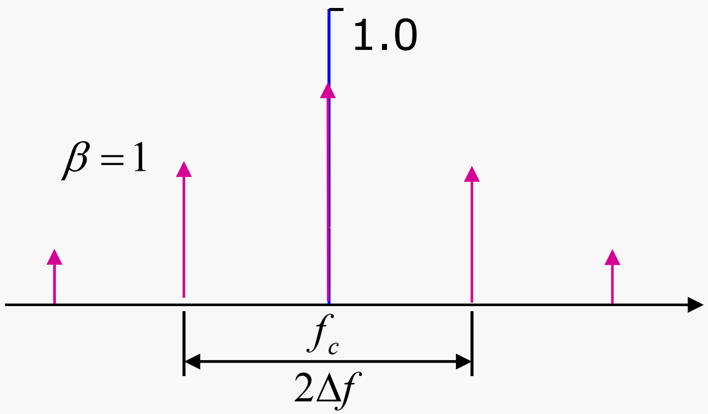
- Goal: to investigate how the amplitude A_m, and frequency f_m, of a sinusoidal modulating wave affect the spectrum of FM wave
- Fixed f_m and varying $A_m \Rightarrow$ frequency deviation $\Delta f = k_f A_m$ and modulation index $\beta = \Delta f / f_m$ are varied



- Increasing A_m increases the number of harmonics in the bandwidth

- Fixed A_m and varying $f_m \Rightarrow \Delta f$ is fixed, but β is varied





  - Increasing f_m decreases the number of harmonics but at the same time increases the spacing between the harmonics.

Effective Bandwidth of FW Waves

- Theoretically, FM bandwidth = infinite

- In practice, for a single tone FM wave, when β is large, B is only slightly greater than the total frequency excursion 2Δf. when β is small, the spectrum is effectively limited to $[f_c - f_m,\ f_c + f_m]$

- Carson's Rule approximation for single-tone modulating wave of frequency f_m

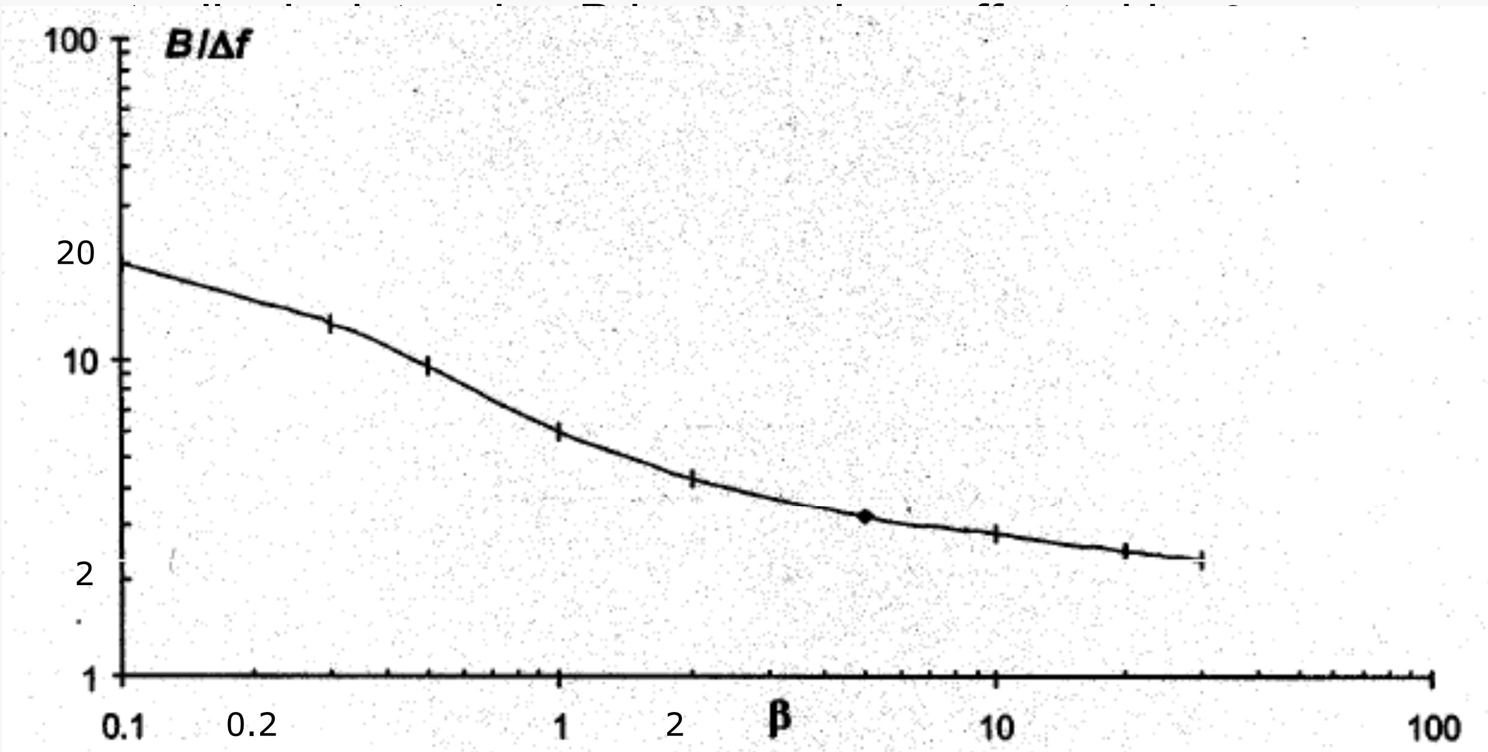
$$B \approx 2\Delta f + 2 f_m = 2(1+\beta) f_m$$

- 99% bandwidth approximation
  - The separation between the two frequencies beyond which none of the side-frequencies is greater than 1% of the unmodulated carrier amplitude
  - i.e $B \approx 2n_{\max} f_m$ where $n_{\max}$ is the max n that satisfies

$$|J_n(\beta)| > 0.01$$

β	0.1	0.3	0.5	1.0	2.0	5.0	10	20	30
$2n_{max}$	2	4	4	6	8	16	28	50	70

- A universal curve for evaluating the 99% bandwidth
  - As β increases, the bandwidth occupied by the significant side-frequencies drops toward that over which the carrier frequency



FM by an Arbitrary Message

- Consider an *arbitrary* m(t) with highest freq component W
- Define deviation ratio D = Δf / W, where $\Delta f = k_f \max|m(t)|$

  D $\Leftrightarrow$ β and W $\Leftrightarrow$ f_m
- Carson's rule applies as

$$B \approx 2\Delta f + 2W = 2W(1 + D)$$

- Carson's rule somewhat underestimate the FM bandwidth requirement, while universal curve yields a somewhat conservative result
- Assess FM bandwidth between the bounds given by Carson's rule and the universal curve

Example

- In north America, the maximum value of frequency deviation Δf is fixed at 75kHz for commercial FM broadcasting by ratio.
- If we take the modulation frequency $W = 15$kHz, which is typically the maximum audio frequency of interest in FM transmission, the corresponding value of the deviation ratio is $D = 75/15 = 5$
- Using *Carson's rule*, the approximate value of the transmission bandwidth of the FM wave is

$$B = 2\,(75+15) = 180\text{kHz}$$

- Using *universal curve*,

$$B = 3.2\,\Delta f = 3.2 \times 75 = 240\text{kHz}$$

Exercise

- Assuming that $m(t) = 10\text{sinc}\left(10^4 t\right)$, determine the transmission bandwidth of an FM modulated signal with $k_f = 4000$

Generation of FM waves

- Direct approach
  - Design an oscillator whose frequency changes with the input voltage => voltage-controlled oscillator (VCO)
- Indirect approach
  - First generate a narrowband FM signal first and then change it to a wideband single
  - Due to the similarity of conventional AM signals, the generation of a narrowband FM signal is straightforward.

Generation of Narrow-band FM

- Consider a narrow band FM wave

$$s_1(t) = A_1 \cos\left[2\pi f_1 t + \phi_1(t)\right]$$

where $\phi_1(t) = 2\pi k_1 \int_0^t m(\tau)d\tau$

f_1 = carrier frequency

k_1 = frequency sensitivity

- Given ϕ_1(t) <<1 with β ≤ 0.3, we may use

$$\begin{cases} \cos\left[\phi_1(t)\right] \approx 1 \\ \sin\left[\phi_1(t)\right] \approx \phi_1(t) \end{cases}$$

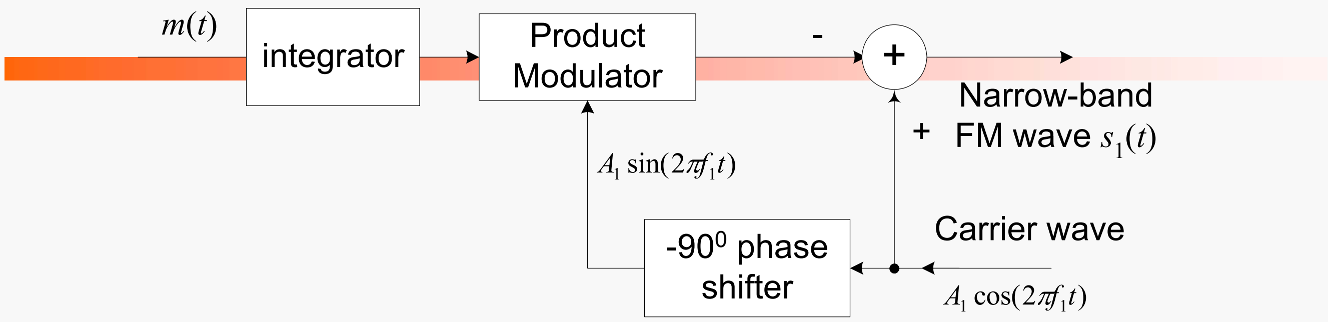
- Correspondingly, we may approximate $s_1(t)$ as

$$s_1(t) = A_1 \cos\left(2\pi f_1 t\right) - A_1 \sin\left(2\pi f_1 t\right)\phi_1(t)$$

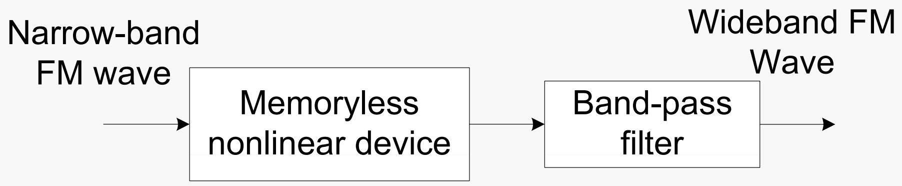
$$= A_1 \cos\left(2\pi f_1 t\right) - 2\pi k_1 A_1 \sin\left(2\pi f_1 t\right)\int_0^t m(\tau)d\tau$$

Narrow-band FW wave

- Narrow-band frequency modulator



- Next, pass $s_1(t)$ through a frequency multiplier, which consists of a non-linear device and a bandpass filter.



  - The input-output relationship of the non-linear device is modeled as

$$s_2(t) = a_1 s_1(t) + a_2 s_1^2(t) + \ldots + a_n s_1^n(t)$$

  - The BPF is used to Pass the FM wave centred at nf_1 and with deviation $n\Delta f_1$ and suppress all other FM spectra

Example: frequency multiplier with n = 2

- Problem: Consider a square-law device based frequency multiplier

$$s_2(t) = a_1 s_1(t) + a_2 s_1^2(t)$$

with

$$s_1(t) = A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau)d\tau\right)$$

- Specify the midband freq. and bandwidth of BPF used in the freq. multiplier for the resulting freq. deviation to be twice that at the input of the nonlinear device
- Solution:

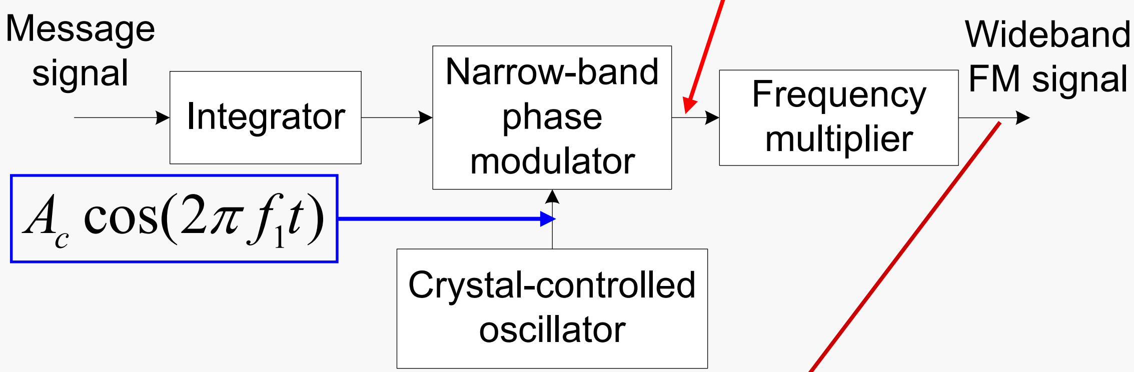
$$s_2(t) = a_1 A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau)d\tau\right) + a_2 A_1^2 \cos^2\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau)d\tau\right)$$

$$= \underbrace{a_1 A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau)d\tau\right) + \frac{a_2 A_1^2}{2}} + \frac{a_2 A_1^2}{2} \cos\left(4\pi f_1 t + 4\pi k_1 \int_0^t m(\tau)d\tau\right)$$

Removed by BPF with $f_c = 2f_1$, $BW > 2\Delta f = 4\Delta f_1$

- Thus, connecting the narrow-band frequency modulator and the frequency multiplier, we may build the wideband frequency modulator

$$s_1(t) = A_1 \cos\left(2\pi f_1 t + 2\pi k_1 \int_0^t m(\tau)d\tau\right)$$



$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

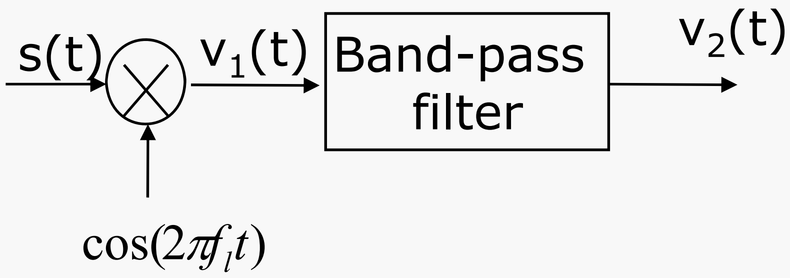
$$f_c = nf_1$$

$$k_f = nk_1$$

$$\Delta f = n\Delta f_1$$

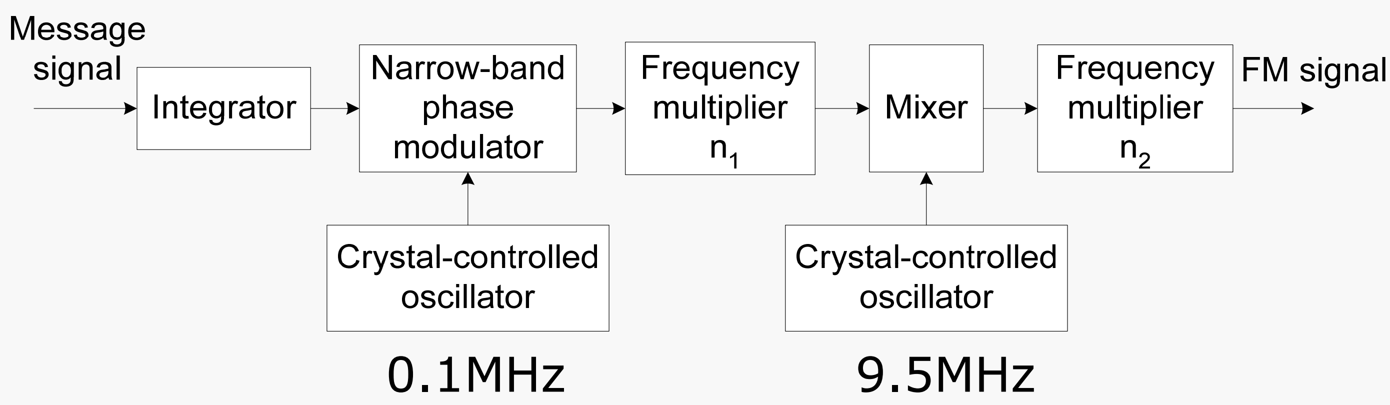
Mixer

- $f_c = nf_1$ may not be the desired carrier frequency. The modulator performs an up/down conversion to shift the modulated signal to the desired center freq.
- This consists of a mixer and a BPF



Exercise: A typical FM transmitter

- Problem: Given the simplified block diagram of a typical FM transmitter used to transmit audio signals containing frequencies in the range 100Hz to 15kHz.
- Desired FM wave: f_c = 100MHz, Δf = 75kHz.
- Set β_1 = 0.2 in the narrowband phase modulation to limit harmonic distortion.
- Specify the two-stage frequency multiplier factors n_1 and n_2



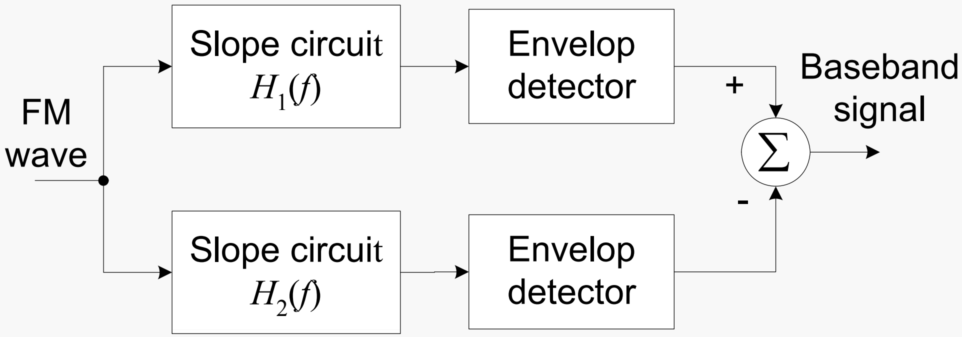
Demodulation of FM
Balanced Frequency Discriminator

- Given FM wave $s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$

$$\frac{d}{dt}s(t) = -A_c\left[2\pi f_c t + 2\pi k_f m(t)\right]\sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

⇨ Hybrid-modulated wave with AM and FM

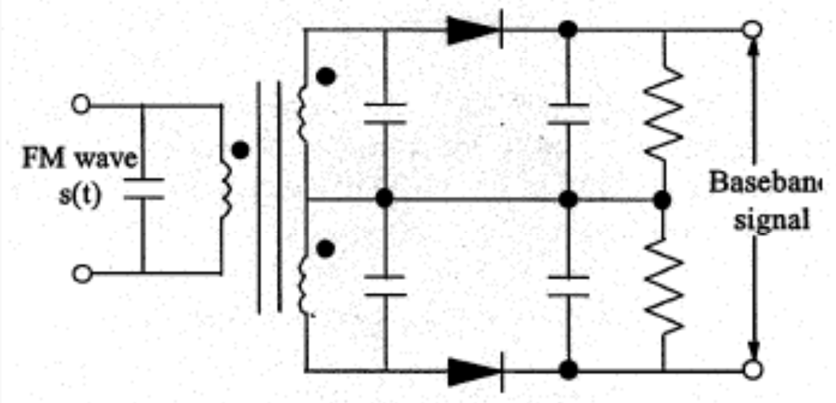
- Differentiator + envelop detector = FM demodulator
- Frequency discriminator: a “freq to amplitude” transform device

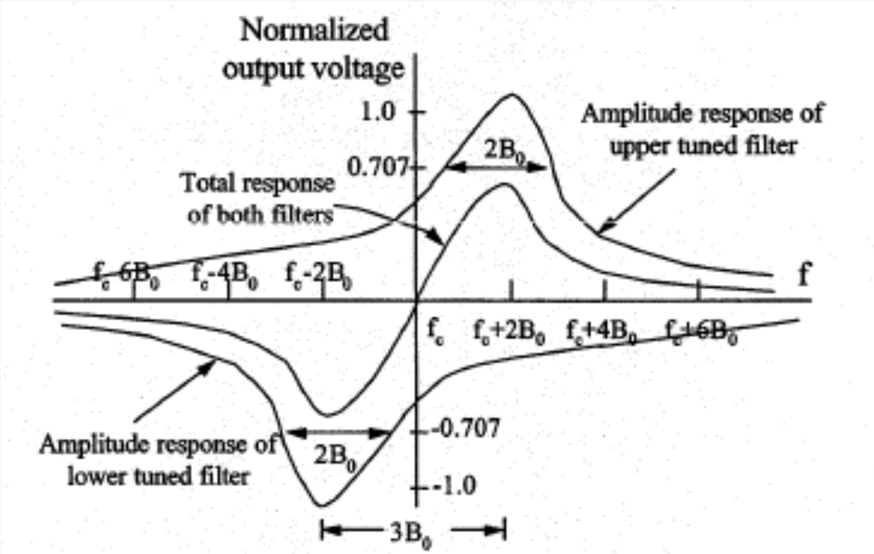


$$H_1(f) = \begin{cases} j2\pi a(f - f_c + B/2), & f_c - B/2 \le f \le f_c + B/2 \\ j2\pi a(f + f_c - B/2), & -f_c - B/2 \le f \le -f_c + B/2 \\ 0, & \text{elsewhere} \end{cases}$$

$$H_2(f) = H_1(-f)$$

Circuit diagram and frequency response





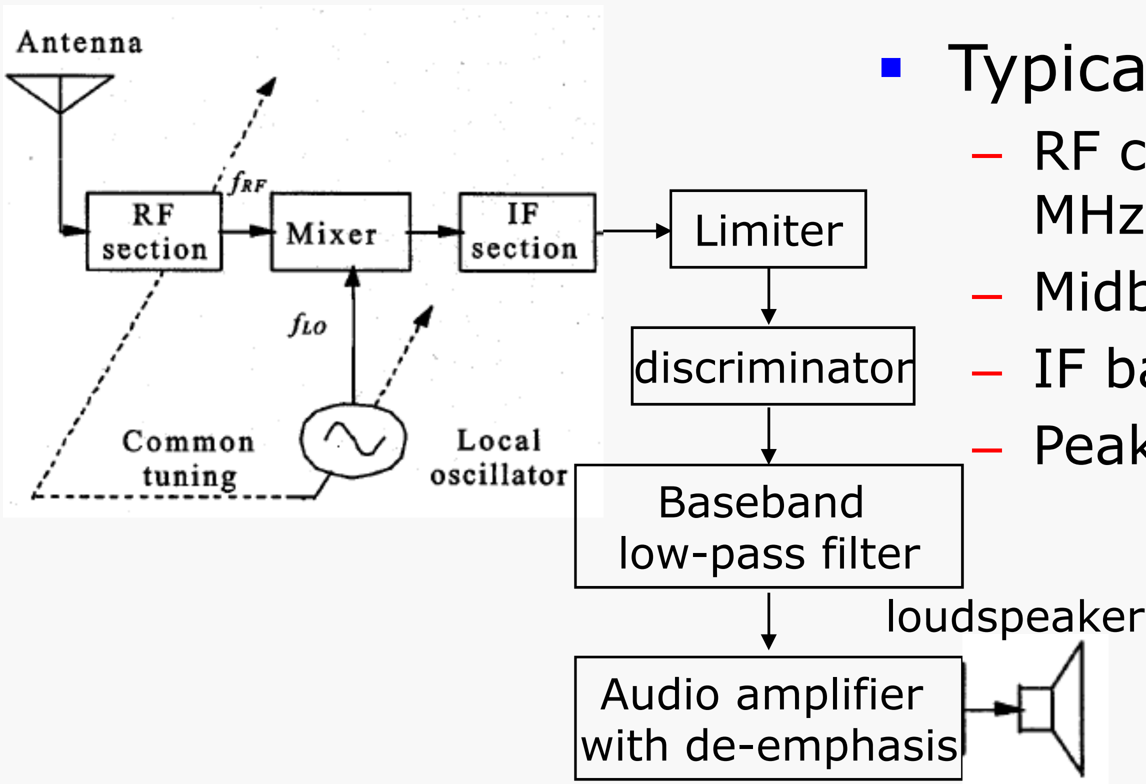
Think …

- Compared with amplitude modulation, angle modulation requires a higher implementation complexity and a higher bandwidth occupancy.

- What is the usefulness of angle modulation systems?

Application: FM Radio broadcasting

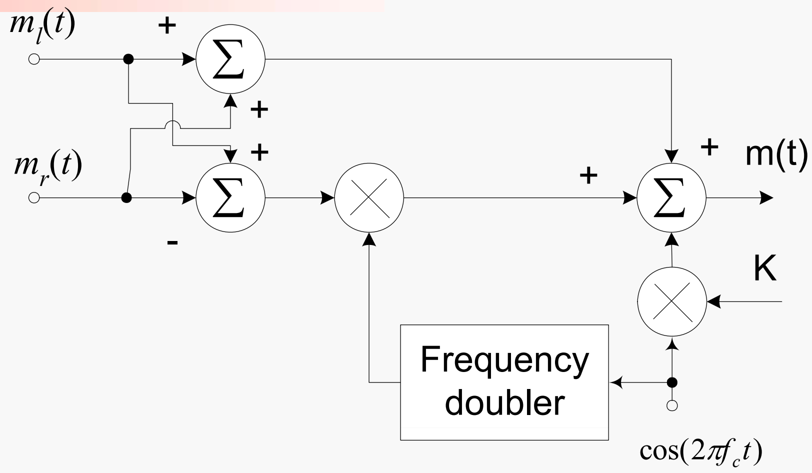
- As with standard AM radio, most FM radio receivers are of super-heterodyne type



- Typical freq parameters
  - RF carrier range = 88~108 MHz
  - Midband of IF = 10.7MHz
  - IF bandwidth = 200kHz
  - Peak freq. deviation = 75KHz

FM Radio Stereo Multiplexing

- Stereo multiplexing is a form of FDM designed to transmit two separate signals via the same carrier.
- Widely used in FM broadcasting to send two different elements of a program (e.g. vocalist and accompanist in an orchestra) so as to give a spatial dimension to its perception by a listener at the receiving end

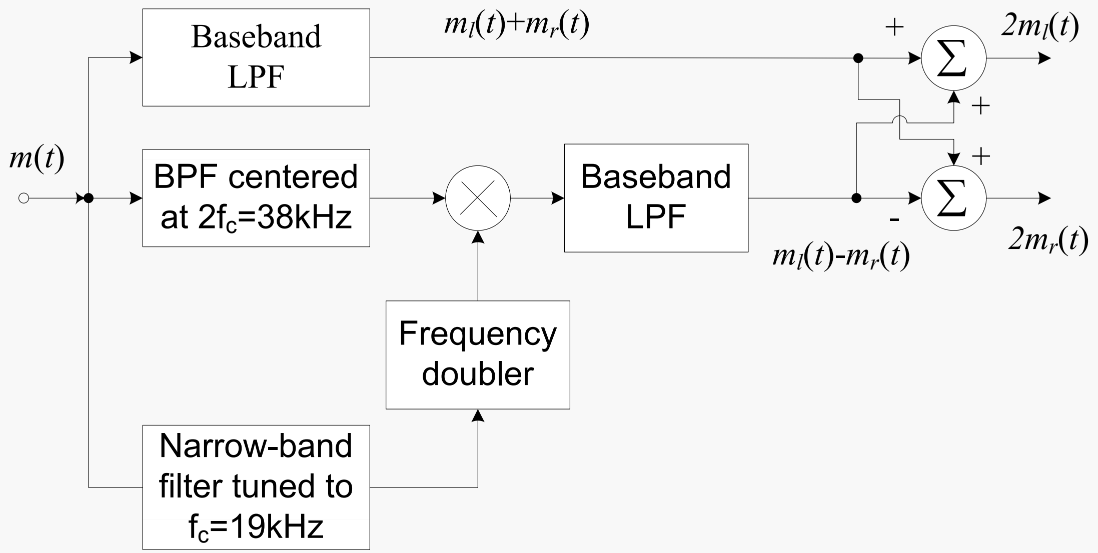


$$m(t) = [m_l(t) + m_r(t)] + [m_l(t) - m_r(t)]\cos(4\pi f_c t) + K\cos(2\pi f_c t)$$

f_c = 19kHz

- The sum signal is left unprocessed in its baseband form
- The difference signal and a 38-kHz subcarrier produce a DSBSC wave
- The 19-kHz pilot is included as a reference for coherent detection

FM-Stereo Receiver



To two loudspeakers

Comparison of Analog-Modulation

- Bandwidth efficiency
  - SSB is the most bandwidth efficient, but cannot effectively transmit DC
  - VSB is a good compromise
  - PM/FM are the least favorable systems
- Power efficiency
  - FM provides high noise immunity
  - Conventional AM is the least power efficient
- Ease of implementation (transmitter and receiver)
  - The simplest receiver structure is conventional AM
  - FM receivers are also easy to implement
  - DSB-SC and SSB-SC requires coherent detector and hence is much more complicated.

Applications

- SSB-SC:
  - Voice transmission over microwave and satellite links
- VSB-SC
  - Widely used in TV broadcasting
- FM
  - High-fidelity radio broadcasting
- Conventional AM
  - AM radio broadcasting
- DSB-SC
  - Hardly used in analog signal transmission!