Principles of Communications

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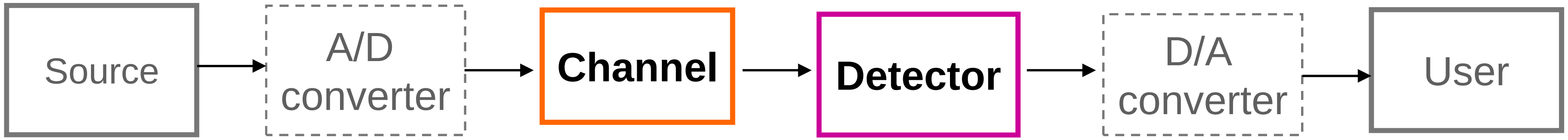
Chapter 5: Digital Transmission through Baseband Channels

Textbook: Ch 10.1-10.5

Topics to be Covered



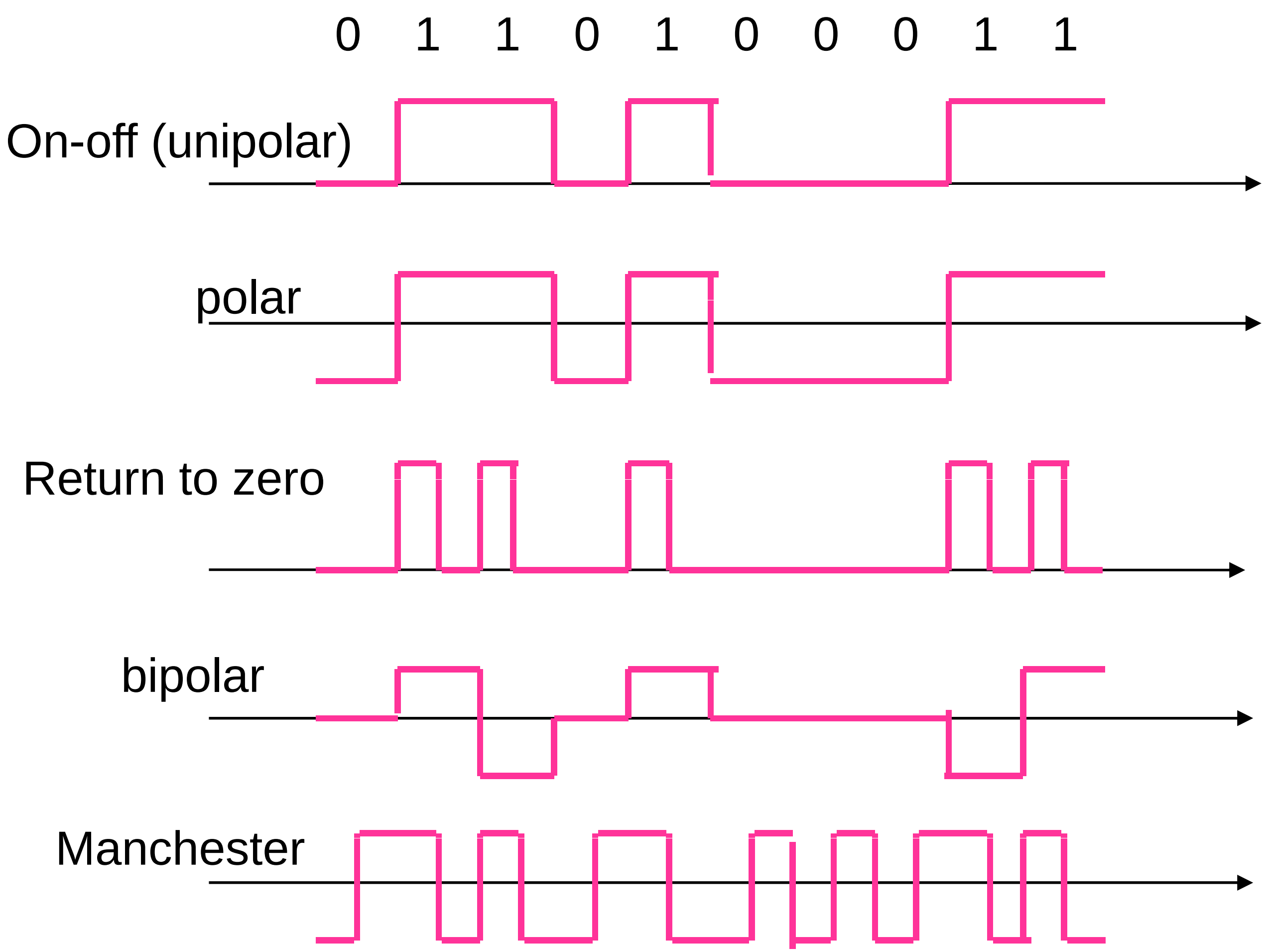
- Digital waveforms over baseband channels
- Band-limited channel and Inter-symbol interference
- Signal design for band-limited channels
- System design and channel equalization

5.1 Baseband Signalling Waveforms

- To send the encoded digital data over a baseband channel, we require the use of format or waveform for representing the data
- System requirement on digital waveforms
  - Easy to synchronize
  - High spectrum utilization efficiency
  - Good noise immunity
  - No dc component and little low frequency component
  - Self-error-correction capability
  - …

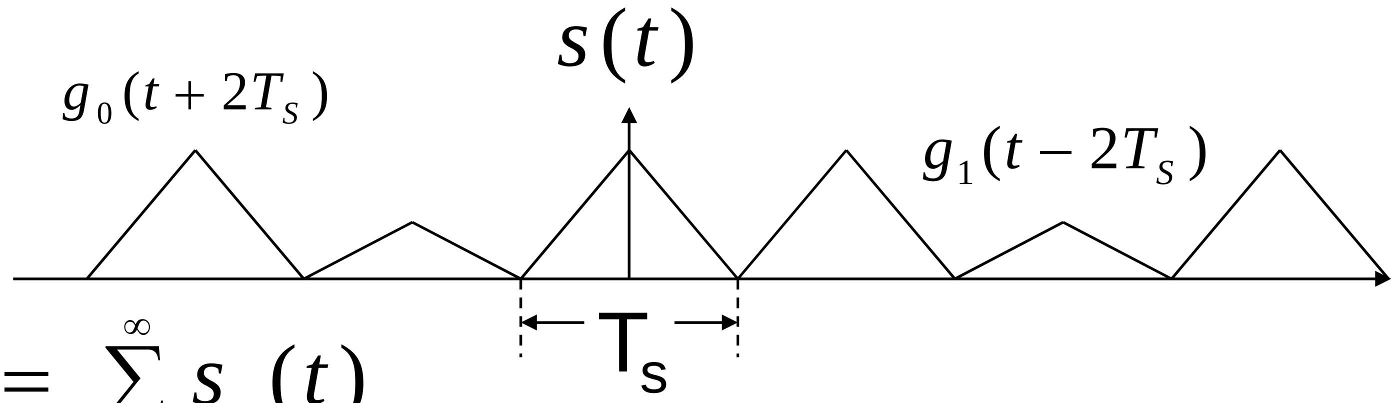
Basic Waveforms

- Many formats available. Some examples:
  - On-off or unipolar signaling
  - Polar signaling
  - Return-to-zero signaling
  - Bipolar signaling – useful because no dc
  - Split-phase or Manchester code – no dc


0 1 1 0 1 0 0 0 1 1
On-off (unipolar)
polar
Return to zero
bipolar
Manchester

Spectra of Baseband Signals

- Consider a random binary sequence $g_0(t)$ - 0，$g_1(t)$ - 1
- The pulses $g_0(t)$ $g_1(t)$ occur independently with probabilities given by p and 1-p，respectively. The duration of each pulse is given by Ts.



$$s(t) = \sum_{n=-\infty}^{\infty} s_n(t)$$

$$s_n(t) = \begin{cases} g_0(t - nT_s), & \text{with prob. } P \\ g_1(t - nT_s), & \text{with prob. } 1 - P \end{cases}$$

Power Spectral Density

- PSD of the baseband signal s(t) is

$$S(f)=\frac{1}{T_s}p(1-p)\left|G_0(f)-G_1(f)\right|^2+\frac{1}{T_s^{\,2}}\sum_{m=-\infty}^{\infty}\left|pG_0(\frac{m}{T_s})+(1-p)G_1(\frac{m}{T_s})\right|^2\delta(f-\frac{m}{T_s})$$

  - 1st term is the continuous freq. component
  - 2nd term is the discrete freq. component

- For polar signalling with $g_0(t) = -g_1(t) = g(t)$ and p=1/2

$$S(f) = \frac{1}{T}|G(f)|^2$$

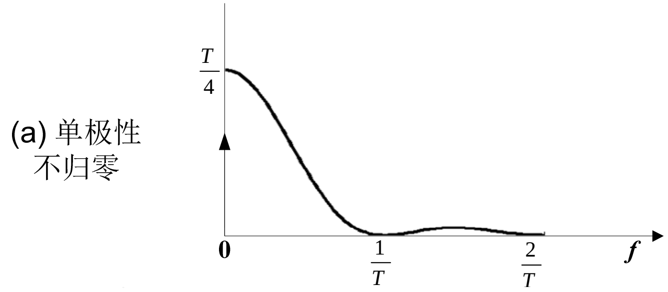
- For unipolar signalling with $g_0(t) = 0$ $g_1(t) = g(t)$ and p=1/2, and g(t) is a rectangular pulse

$$G(f) = T\left[\frac{\sin \pi fT}{\pi fT}\right] \Rightarrow S_x(f) = \frac{T}{4}\left[\frac{\sin \pi fT}{\pi fT}\right]^2 + \frac{1}{4}\delta(f)$$

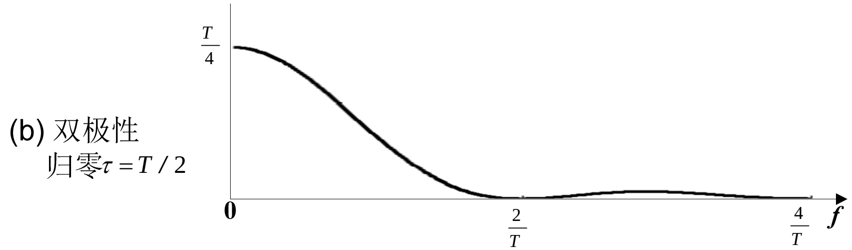
- For return-to-zero unipolar signalling $\tau = T/2$

$$S_x(f) = \frac{T}{16}\left[\frac{\sin \pi fT/2}{\pi fT/2}\right]^2 + \frac{1}{16}\delta(f) + \frac{1}{4}\sum_{\text{odd } m}\frac{1}{[m\pi]^2}\delta(f - \frac{m}{T})$$

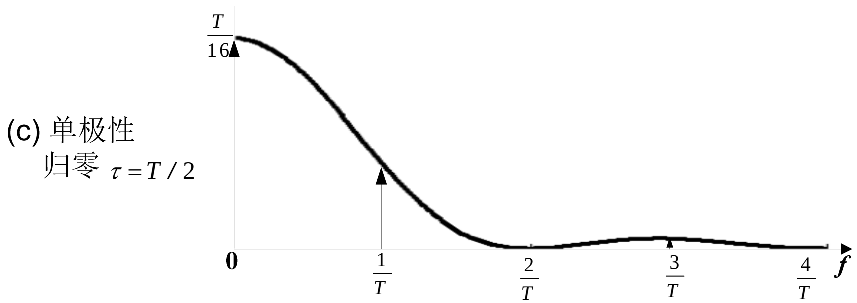
PSD of Basic Waveforms



unipolar



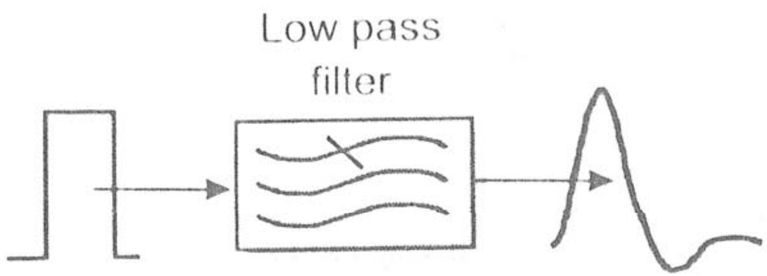
Return-to-zero polar



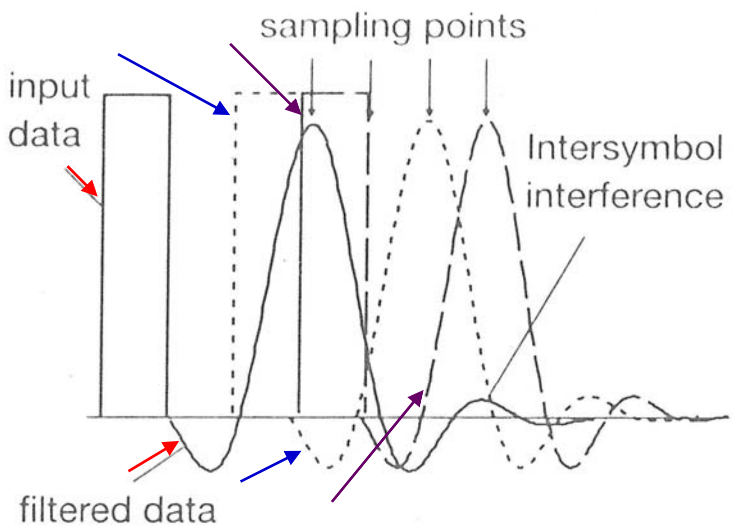
Return-to-zero unipolar

5.2 Bandlimited Channel

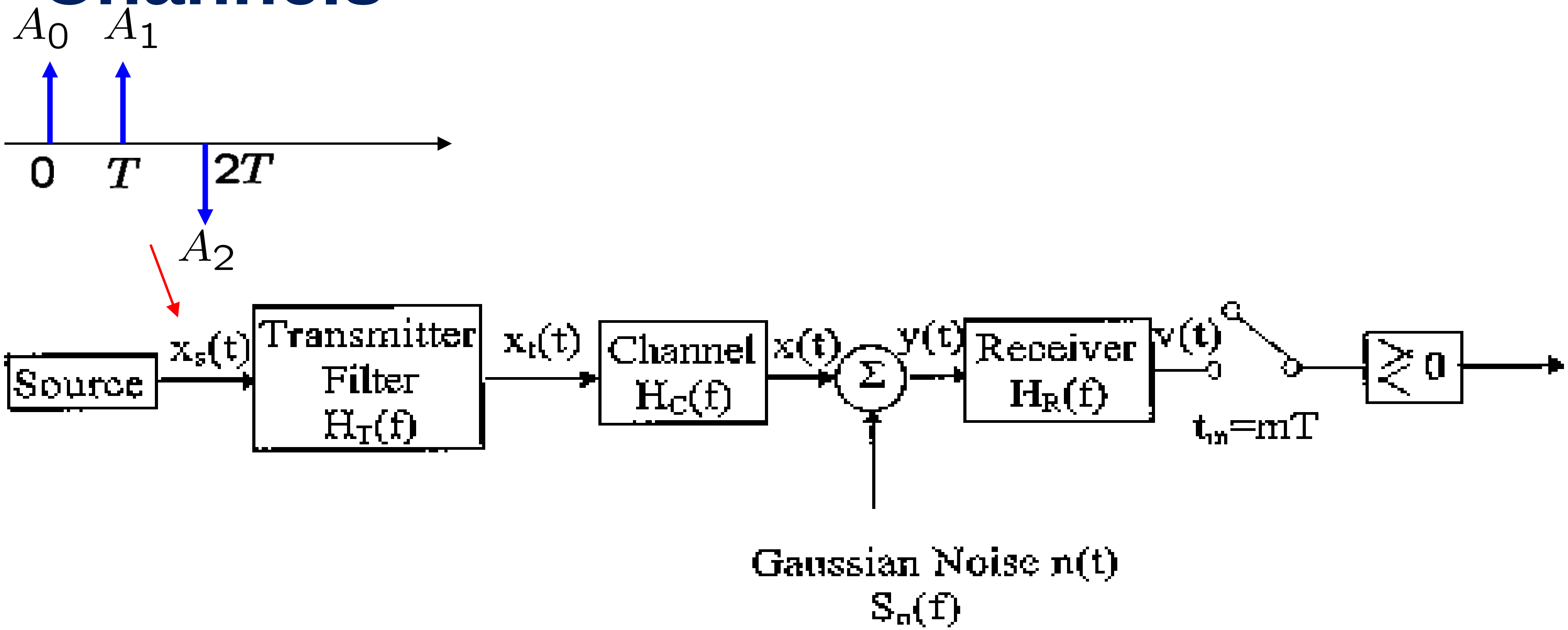
- A bandlimited channel can be modeled as a linear filter with frequency response limited to certain frequency range
- The filtering effect will cause a spreading (or smearing out) of individual data symbols passing through a channel



- For consecutive symbols, this spreading causes part of the symbol energy to overlap with neighbouring symbols, causing intersymbol interference (ISI).



Baseband Signaling through Bandlimited Channels



Input to tx filter

$$x_s(t) = \sum_{i=-\infty}^{\infty} A_i \delta(t - iT)$$

Output of tx filter

$$x_t(t) = \sum_{i=-\infty}^{\infty} A_i h_T(t - iT)$$

Output of rx filter

$$v(t) = x_s(t) * h_T(t) * h_c(t) * h_R(t) + n(t) * h_R(t)$$

- Pulse shape at the receiver filter output

$$p(t) = h_T(t) * h_c(t) * h_R(t)$$

  - Impulse response of the cascade connection of tx, channel, and rx filters

- Overall frequency response

$$P(f) = H_T(f)H_C(f)H_R(f)$$

- Receiving filter output

$$v(t) = \sum_{k=-\infty}^{\infty} A_k p(t - kT) + n_o(t)$$

$$n_o(t) = n(t) * h_R(t)$$

Intersymbol Interference

- The receiving filter output v(t) is sampled at t_m=mT (to detect A_m)

$$v(t_m) = \sum_{k=-\infty}^{\infty} A_k p(mT - kT) + n_o(t_m)$$

$$= \underbrace{A_m p(0)}_{\text{Desired signal}} + \underbrace{\sum_{k \neq m}^{\infty} A_k p[(m-k)T]}_{\text{intersymbol interference (ISI)}} + \underbrace{n_o(t_m)}_{\text{Gaussian noise}}$$

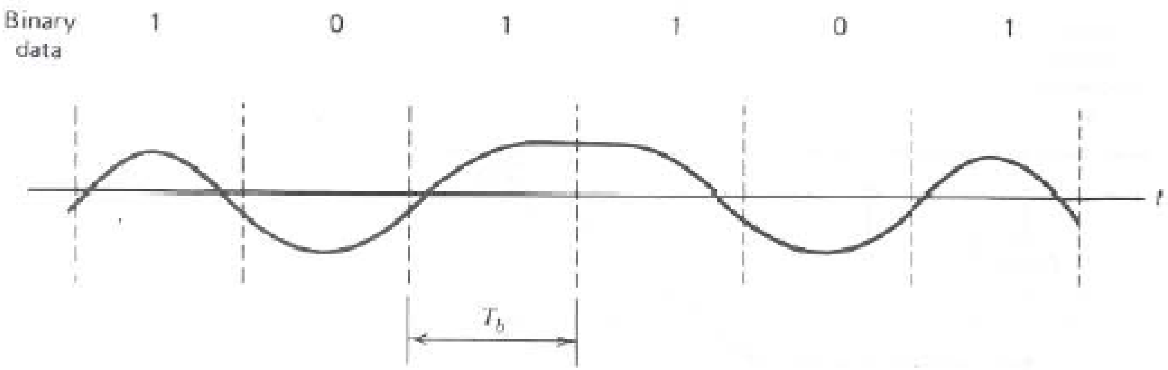
- ISI can significantly degrade the ability of the data detector.

Eye Diagrams

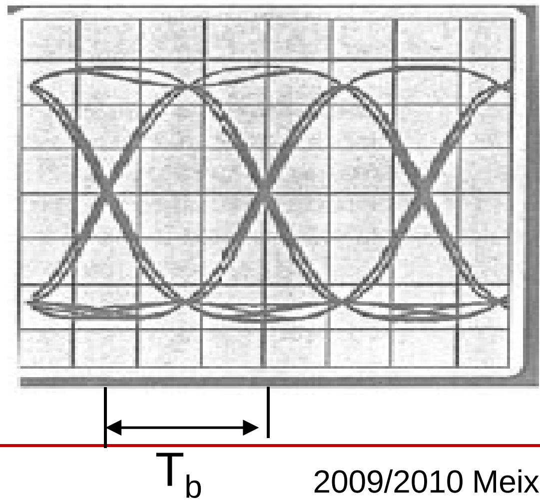
- A visual method to investigate the problem of ISI
- Generated by connecting the received waveforms to a conventional oscilloscope
- Oscilloscope is re-triggered at every symbol period or multiple of symbol periods using a timing recovery signal.
- Segments of the received waveforms are then superimposed on one another
- The resulting display is called an eye pattern

Eye Diagrams (cont'd)

- Distorted binary wave

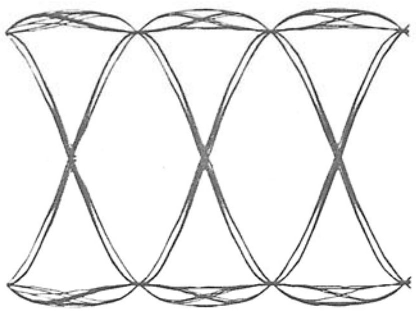


- Eye pattern

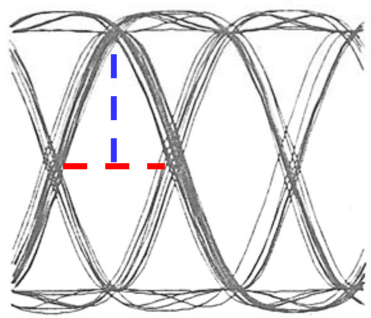


Eye Diagrams (cont'd)

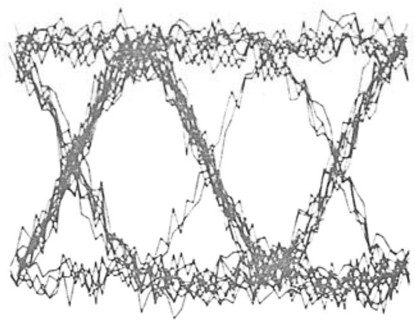
- Example eye diagrams for different distortions, each has a distinctive effect on the appearance of the "eye opening"
  - The width of the eye opening defines the time interval over which the wave can be sampled. The best sampling time is the instant when the eye is open widest
  - The height of the eye opening defines the margin over noise



Signal only



Signal + timing error



Signal + noise

ISI Minimization

- Choose transmitter and receiver filters which shape the received pulse function so as to eliminate or minimize interference between adjacent pulses, hence not to degrade the bit error rate performance of the link

5.3 Signal Design for Bandlimited Channel

Zero ISI

- The effect of ISI can be completely negated if it is possible to obtain a received pulse shape, p(t), such that

$$p(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Echos made to be zero at sampling points

or
$$\sum_{k=-\infty}^{\infty} P\left(f + \frac{k}{T}\right) = \text{constant}$$

- This is the **Nyquist condition for Zero ISI**
- If p(t) satisfies the above condition, the receiver output simplifies to

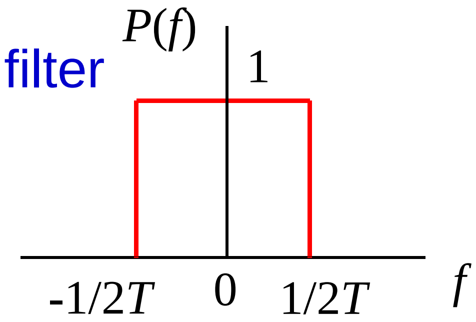
$$v(t_m) = A_m + n_o(t_m)$$

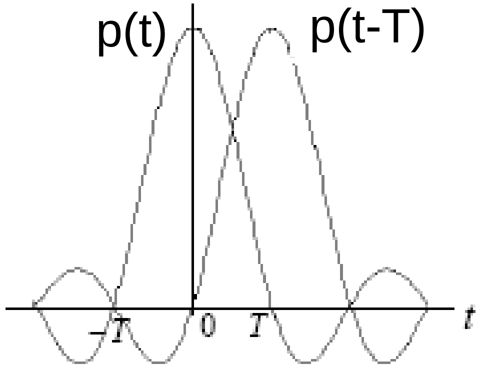
Nyquist Condition: Ideal Solution

- Nyquist's first method for eliminating ISI is to use

$$P(f) = \begin{cases} 1 & |f| < \frac{1}{2T} \\ 0 & \text{otherwise} \end{cases} \quad \Leftrightarrow \quad p(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \text{sinc}\left(\frac{t}{T}\right)$$

"brick wall" filter





Let $B_0 = \frac{1}{2T} = \frac{R_s}{2}$ = called the Nyquist bandwdith,

The minimum transmission bandwidth for zero ISI. A channel with bandwidth B_0 can support a max. transmission rate of $2B_0$ symbols/sec

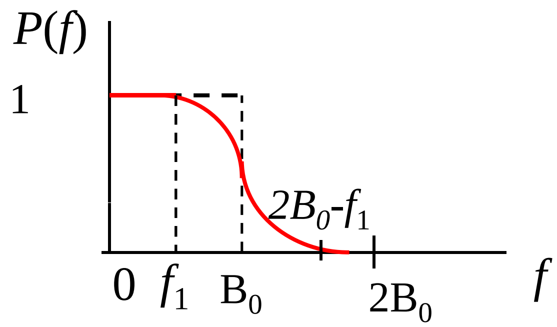
Achieving Nyquist Condition

- Difficult to design such p(t) or P(f)
  - P(f) is physically unrealizable due to the abrupt transitions at $\pm B_0$
  - p(t) decays slowly for large t, resulting in little margin of error in sampling times in the receiver.
  - This demands accurate sample point timing - a major challenge in modem / data receiver design.
  - Inaccuracy in symbol timing is referred to as timing jitter.

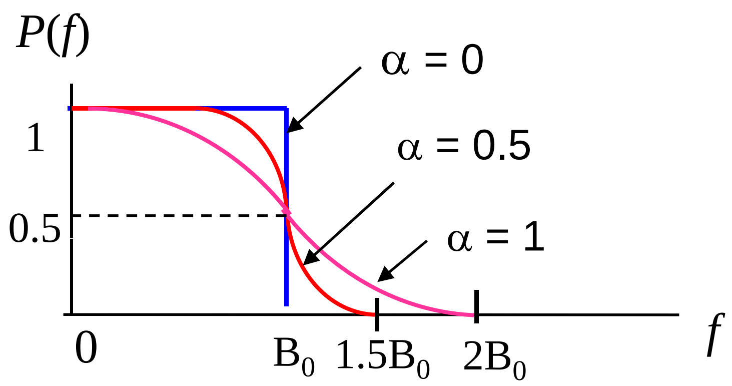
Practical Solution: Raised Cosine Spectrum

- Let P(f) decrease toward to zero gradually rather than abruptly.
- P(f) is made up of 3 parts: passband, stopband, and transition band. The transition band is shaped like a cosine wave.

$$P(f)=\begin{cases} 1 & 0\le |f| < f_1 \\ \frac{1}{2}\left\{1+\cos\left[\frac{\pi(|f|-f_1)}{2B_0-2f_1}\right]\right\} & f_1\le |f| < 2B_0-f_1 \\ 0 & |f|\ge 2B_0-f_1 \end{cases}$$



Raised Cosine Spectrum



Roll-off factor

$$\alpha = 1 - \frac{f_1}{B_0}$$

- The sharpness of the filter is controlled by α . When $\alpha = 0$ this reduces to the “brick wall” filter.
- The bandwidth required by using raised cosine spectrum increased from its minimum value B_0 to actual bandwidth $B = B_0(1+\alpha)$

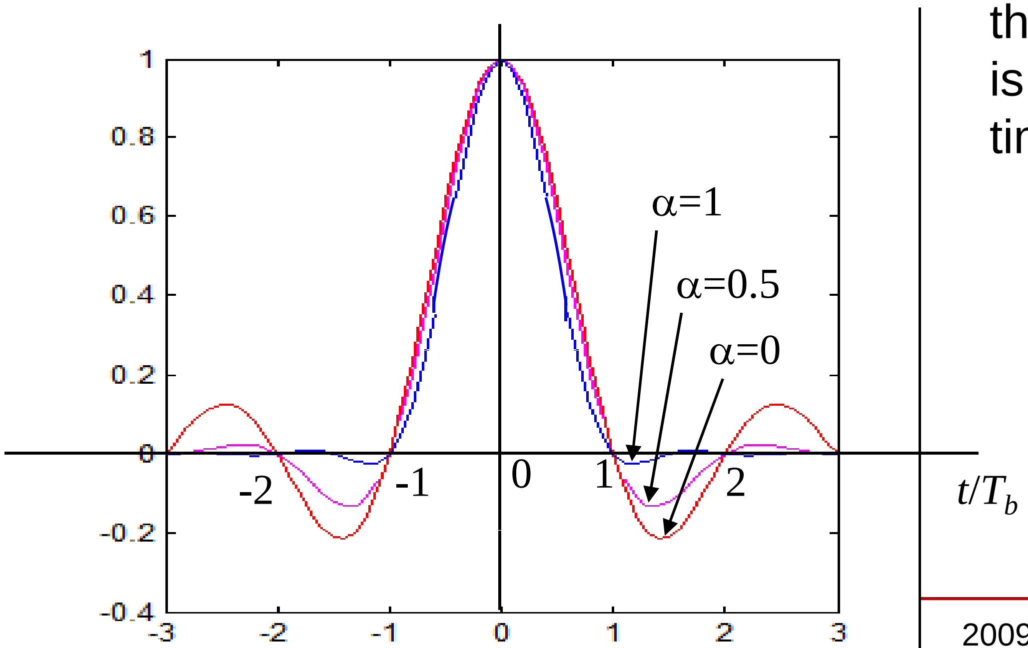
Time-Domain Pulse Shape

- Inverse Fourier transform of raised cosine spectrum

$$p(t) = \mathrm{sinc}(2B_0 t)\frac{\cos(2\pi\alpha B_0 t)}{1-16\alpha^2 B_0^2 t^2}$$

Ensures zero crossing at desired sampling instants

Decreases as $1/t^2$, reduces the tail of the pulses such that the data receiving is relatively insensitive to sampling time error



Choice of Roll-off Factor

- Benefits of small a
  - Higher bandwidth efficiency
- Benefits of large a
  - simpler filter with fewer stages hence easier to implement
  - less sensitive to symbol timing accuracy

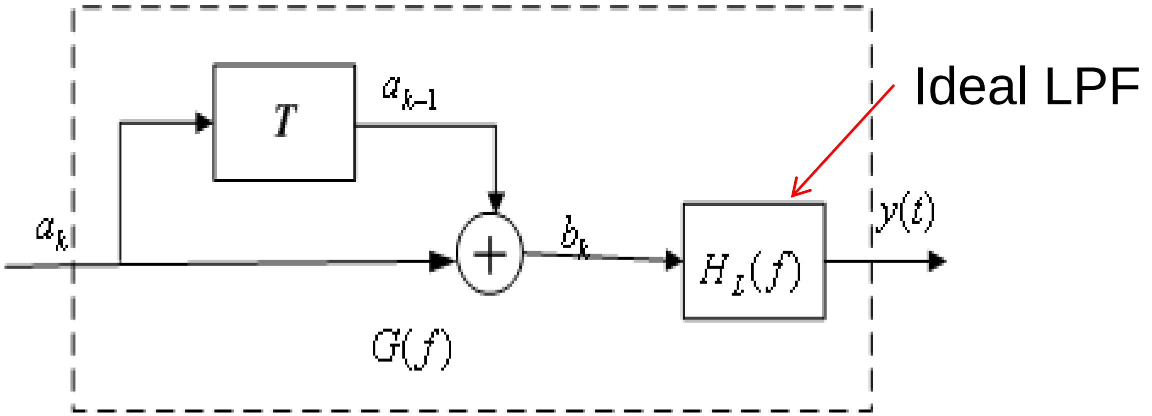
Signal Design with Controlled ISI

Partial Response Signals

- Relax the condition of zero ISI and allow a controlled amount of ISI
- Then we can achieve the max. symbol rate of 2W symbols/sec
- The ISI we introduce is deterministic or “controlled”; hence it can be taken into account at the receiver

Duobinary Signal

- Let {ak} be the binary sequence to be transmitted. The pulse duration is T.
- Two adjacent pulses are added together, i.e. $b_k = a_k + a_{k-1}$



(a)

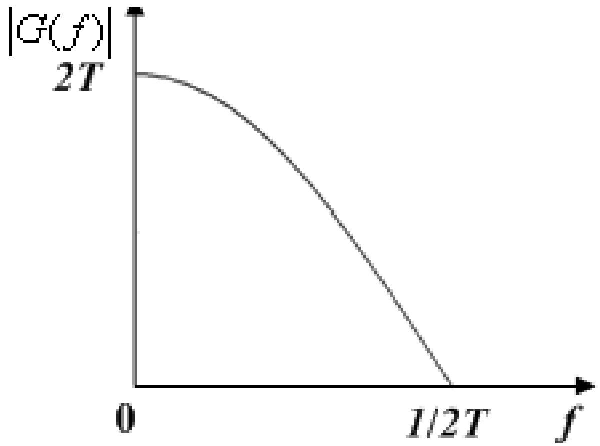
- The resulting sequence $\{b_k\}$ is called duobinary signal

Characteristics of Duobinary Signal

Frequency domain

$$G(f) = \left(1 + e^{-j2\pi fT}\right) H_L(f) \qquad H_L(f) = \begin{cases} T & (|f| \le 1/2T) \\ 0 & (\text{otherwise}) \end{cases}$$

$$= \begin{cases} 2Te^{-j\pi fT} \cos \pi fT & (|f| \le 1/2T) \\ 0 & (\text{otherwise}) \end{cases}$$



Time domain Characteristics

$$g(t)=[\delta(t)+\delta(t-T)]*h_L(t) \quad =\frac{\sin \pi t/T}{\pi t/T}+\frac{\sin \pi(t-T)/T}{\pi(t-T)/T}$$

$$=\text{sinc}\left(\frac{t}{T}\right)+\text{sinc}\left(\frac{t-T}{T}\right) \quad =\frac{T^2}{\pi t}\cdot\frac{\sin \pi t/T}{(T-t)}$$

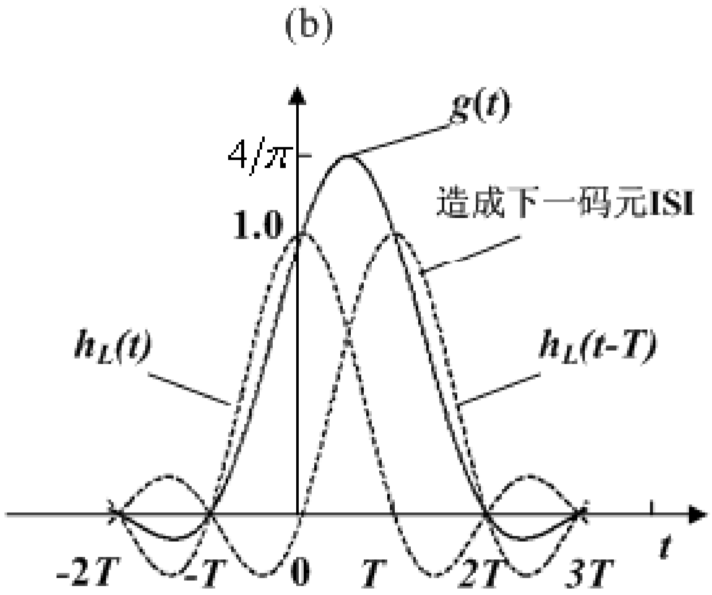
- $g(t)$ is called a **duobinary signal pulse**
- It is observed that:

$g(0)=g_0=1$ (The current symbol)

$g(T)=g_1=1$ (ISI to the next symbol)

$g(iT)=g_i=0 \quad (i \neq 0,1)$

Decays as $1/t^2$, and spectrum within 1/2T



Decoding

- Without noise, the received signal is the same as the transmitted signal

$$y_k = \sum_{i=0}^{\infty} a_i g_{k-i} = a_k + a_{k-1} = b_k$$

A 3-level sequence

- When $\{a_k\}$ is a polar sequence with values ＋1 or －1:

$$y_k = b_k = \begin{cases} 2 & (a_k = a_{k-1} = 1) \\ 0 & (a_k = 1, a_{k-1} = -1 \text{ or } a_k = -1, a_{k-1} = 1) \\ -2 & (a_k = a_{k-1} = -1) \end{cases}$$

- When $\{a_k\}$ is a unipolar sequence with values 0 or 1

$$y_k = b_k = \begin{cases} 0 & (a_k = a_{k-1} = 0) \\ 1 & (a_k = 0, a_{k-1} = 1 \text{ or } a_k = 1, a_{k-1} = 0) \\ 2 & (a_k = a_{k-1} = 1) \end{cases}$$

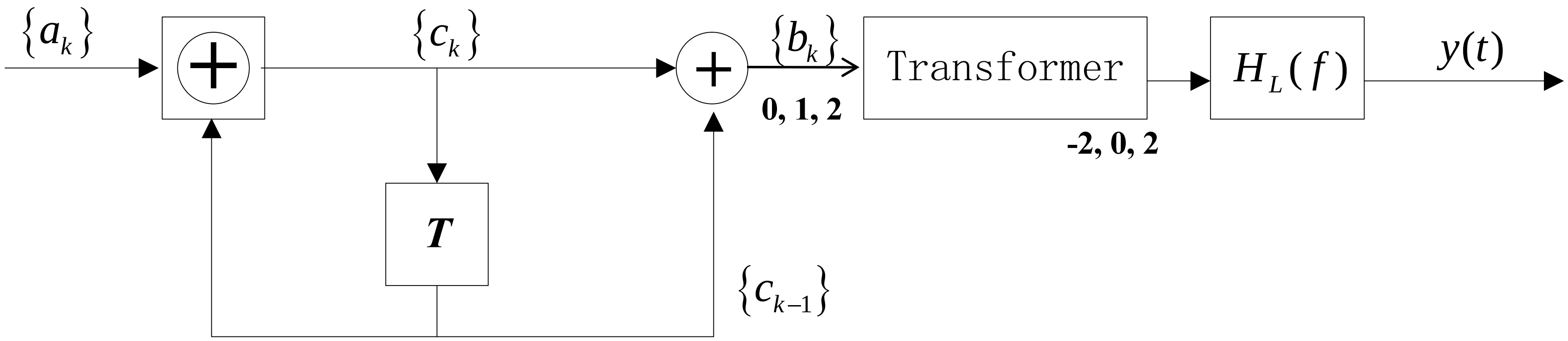
- To recover the transmitted sequence, we can use

$$\hat{a}_k = b_k - \hat{a}_{k-1} = y_k - \hat{a}_{k-1}$$

  - Main drawback: the detection of the current symbol relies on the detection of the previous symbol => error propagation will occur
- How to solve the ambiguity problem and error propagation?
- Precoding:
  - Apply differential encoding on $\{a_k\}$ so that $c_k = a_k \oplus c_{k-1}$
  - Then the output of the duobinary signal system is

$$b_k = c_k + c_{k-1}$$

Block Diagram of Precoded Duobinary Signal



Modified Duobinary Signal

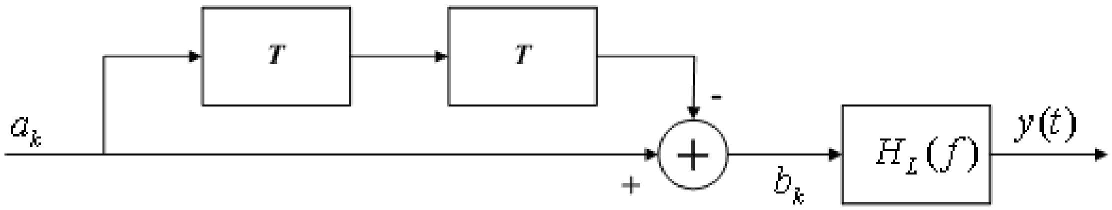
- Modified duobinary signal

$$b_k = a_k - a_{k-2}$$

- After LPF $H_L(f)$, the overall response is

$$G(f) = (1 - e^{-j4\pi fT}) H_L(f) = \begin{cases} 2Tje^{-j2\pi fT} \sin 2\pi fT & (|f| \le 1/2T) \\ 0 & \text{otherwise} \end{cases}$$

$$g(t) = \frac{\sin \pi t / T}{\pi t / T} - \frac{\sin \pi (t - 2T) / T}{\pi (t - 2T) / T} = -\frac{2T^2 \sin \pi t / T}{\pi t (t - 2T)}$$



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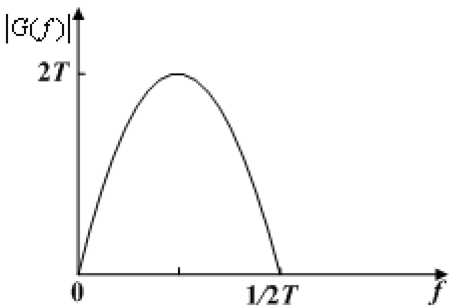

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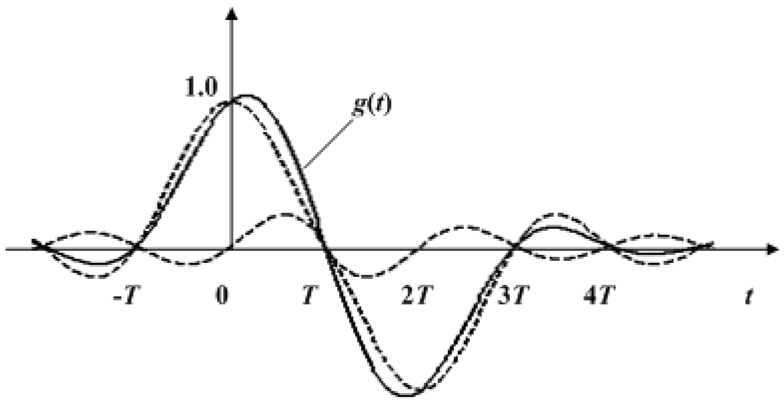


(c)

Properties

- The magnitude spectrum is a half-sin wave and hence easy to implement
- No dc component and small low freq. component
- At sampling interval T, the sampled values are

$$g(0) = g_0 = 1$$
$$g(T) = g_1 = 0$$
$$g(2T) = g_2 = -1$$
$$g(iT) = g_i = 0, i \neq 0,1,2$$

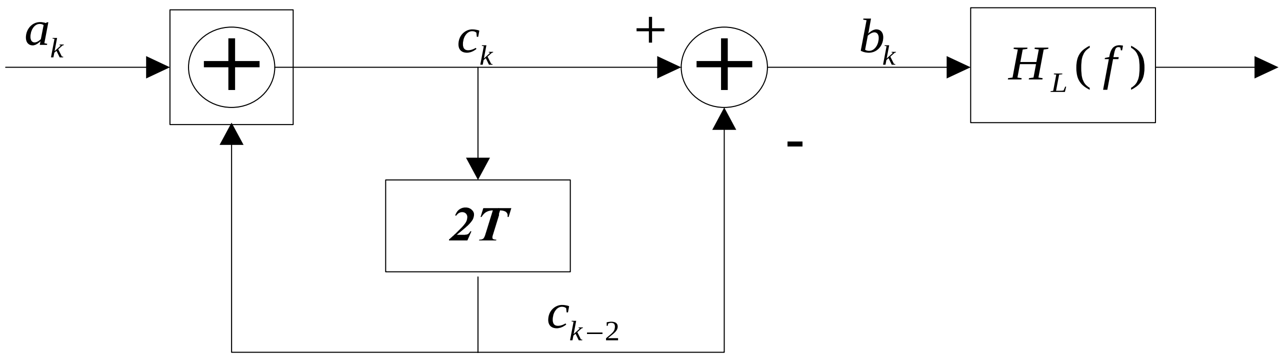
- $g(t)$ also delays as $1/t^2$. But at $t = T$, the timing offset may cause significant problem.

Decoding of modified duobinary signal

- To overcome error propagation, precoding is also needed.

$$c_k = a_k \oplus c_{k-2}$$

- The coded signal is $b_k = c_k - c_{k-2}$



- Now we have discussed:
  - Pulse shapes of baseband signal and their power spectrum
  - ISI in bandlimited channels
  - Signal design for zero ISI and controlled ISI
- We next discuss system design in the presence of channel distortion
  - Optimal transmitting and receiving filters
  - Channel equalizer

5.4 Optimum Transmit/Receiver Filter

- Recall that when zero-ISI condition is satisfied by p(t) with raised cosine spectrum P(f), then the sampled output of the receiver filter is $V_m = A_m + N_m$ (assume $p(0) = 1$)
- Consider binary PAM transmission: $A_m = \pm d$
- Variance of N_m = $\sigma^2 = \int_{-\infty}^{\infty} S_n(f)|H_R(f)|^2 df$

with $P(f) = H_T(f)H_C(f)H_R(f)$ $\quad p(t) = h_T(t) * h_C(t) * h_R(t)$

$$\Longrightarrow \quad P_e = Q\left(\frac{d}{\sigma}\right)$$

Error Probability can be minimized through a proper choice of H_R(f) and H_T(f) so that d/σ is maximum (assuming H_C(f) fixed and P(f) given)

Optimal Solution

- Compensate the channel distortion equally between the transmitter and receiver filters

$$\begin{cases} |H_T(f)| = \dfrac{\sqrt{P(f)}}{|H_c(f)|^{1/2}} \\[2ex] |H_R(f)| = \dfrac{\sqrt{P(f)}}{|H_c(f)|^{1/2}} \end{cases} \qquad \text{for } |f| \le W$$

- Then, the transmit signal energy is given by

$$E_{av} = \int_{-\infty}^{\infty} d^2 h_T^2(t)dt = \int_{-\infty}^{\infty} d^2 H_T^2(f)df = \int_{-W}^{W} \frac{d^2 P(f)}{|H_C(f)|}df$$

By Parseval's theorem

- Hence $$d^2 = E_{av} \cdot \left[\int_{-W}^{W} \frac{P(f)}{|H_C(f)|}df\right]^{-1}$$

- Noise variance at the output of the receive filter is

$$\sigma^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |H_R(f)|^2 df = \frac{N_0}{2}\int_{-W}^{W} \frac{P(f)}{|H_C(f)|} df$$

$$\Rightarrow \quad P_{e,\min} = Q\left[\sqrt{\frac{2E_{av}}{N_0}} \underbrace{\left\{\int_{-W}^{W} \frac{P(f)}{|H_c(f)|} df\right\}^{-1}}\right]$$

Performance loss due to channel distortion

- Special case: $H_C(f) = 1$ for $|f| \leq W$
  - This is the ideal case with "flat" fading
  - No loss, same as the matched filter receiver for AWGN channel

Exercise

- Determine the optimum transmitting and receiving filters for a binary communications system that transmits data at a rate R=1/T = 4800 bps over a channel with a frequency response $|H_c(f)| = \frac{1}{\sqrt{1+(\frac{f}{W})^2}}$; $|f| \leq W$ where W= 4800 Hz
- The additive noise is zero-mean white Gaussian with spectral density $N_0/2 = 10^{-15}$ Watt/Hz

Solution

- Since W = 1/T = 4800, we use a signal pulse with a raised cosine spectrum and a roll-off factor = 1.
- Thus,

$$P(f) = \frac{1}{2}[1+\cos(\pi|f|)] = \cos^2\left(\frac{\pi|f|}{9600}\right)$$

- Therefore

$$|H_T(f)| = |H_R(f)| = \cos\left(\frac{\pi|f|}{9600}\right)\left[1+\left(\frac{f}{4800}\right)^2\right]^{1/4}, \text{for } |f| \leq 4800$$

- One can now use these filters to determine the amount of transmit energy required to achieve a specified error probability

Performance with ISI

- If zero-ISI condition is not met, then

$$V_m = A_m + \sum_{k \neq m} A_k p[(m-k)T] + N_m$$

- Let

$$A_I = \sum_{k \neq m} I_k = \sum_{k \neq m} A_k p[(m-k)T]$$

- Then

$$V_m = A_m + A_I + N_m$$

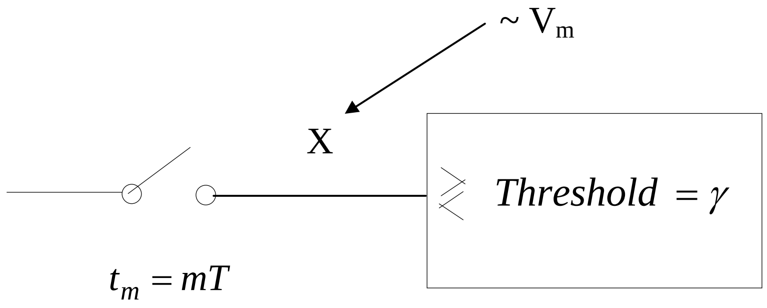
- Often only 2M significant terms are considered. Hence

$$V_m = A_m + A'_I + N_m \quad \text{with} \quad A'_I = \sum_{k=-M}^{M} A_k p[(m-k)T]$$

- Finding the probability of error in this case is quite difficult. Various approximation can be used (Gaussian approximation, Chernoff bound, etc).

- What is the solution?

Monte Carlo Simulation



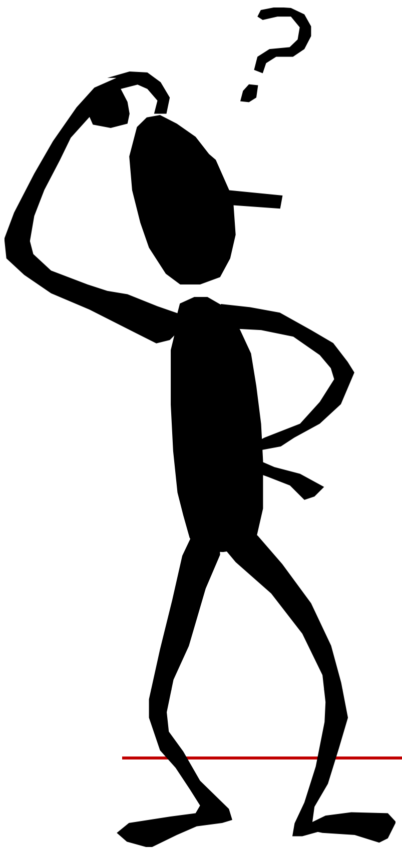
Let

$$I(x) = \begin{cases} 1 & error\ occurs \\ 0 & else \end{cases}$$

$$\therefore \quad P_e = \frac{1}{L}\sum_{l=1}^{L} I\left(X^{(l)}\right)$$

where $X^{(1)}$, $X^{(2)}$, ... , $X^{(L)}$ are i.i.d. (*independent and identically distributed*) random samples

- If one wants P_e to be within 10% accuracy, how many independent simulation runs do we need?
- If $P_e \sim 10^{-9}$ (this is typically the case for optical communication systems), and assume each simulation run takes 1 msec, how long will the simulation take?



- We have shown that
  - By properly designing the transmitting and receiving filters one can guarantee zero ISI at sampling instants, thereby minimizing Pe.
  - Appropriate when the channel is precisely known and its characteristics do not change with time
  - In practice, the channel is unknown or time-varying

- We now consider: channel equalizer
  - A receiving filter with adjustable frequency response
  - With channel measurement, one can adjust the frequency response of the receiving filter so that the overall filter response is near optimum

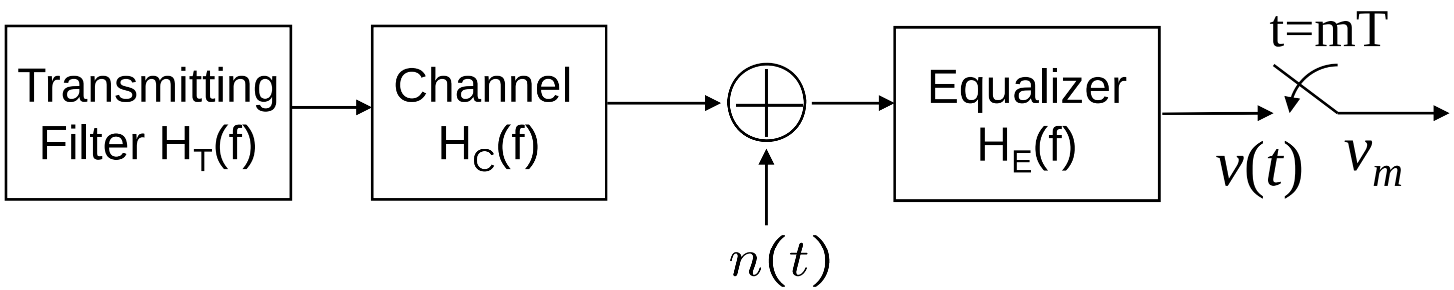
5.5 Equalizer

- Two main types of equalizers
  - Preset equalizers
  - Adaptive equalizers
- Preset equalizers
  - For channels whose frequency response characteristics are unknown but time-invariant
  - We may measure the channel characteristics, adjust the parameters of the equalizer
  - Once adjusted, the equalizer parameters remain fixed during the transmission of data
  - Such equalizers are called preset equalizers

Equalizer (cont'd)

- Adaptive equalizers
  - Update their parameters on a periodic basis during the transmission of data
  - This is often done by sending a known signal through the channel and allowing the equalizer to adjust its parameters in response to this known signal (which is known as Training sequence)
  - Adaptive equalizers are useful when the channel characteristics are unknown or if they change slowly with time.

Equalizer Configuration



- Overall frequency response:

$$H_o(f) = H_T(f)H_C(f)H_E(f)$$

- To guarantee zero ISI, Nyquist criterion must be satisfied

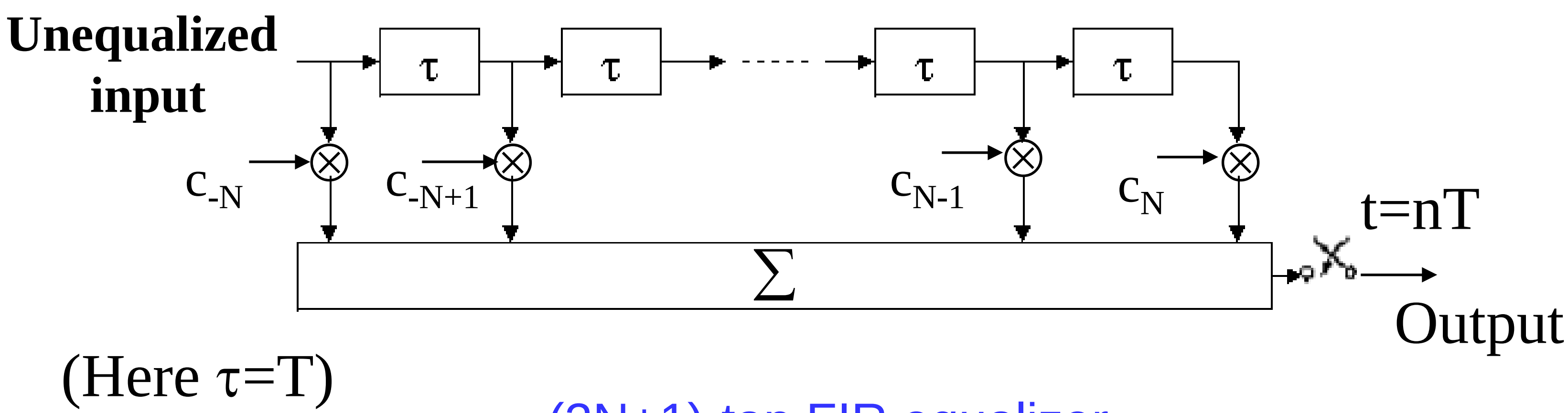
$$\sum_{k=-\infty}^{\infty} H_o\left(f + \frac{k}{T}\right) = \text{constant}$$

- Ideal zero-ISI equalizer is an inverse channel filter with

$$H_E(f) \propto \frac{1}{H_T(f)H_C(f)} \quad |f| \leq 1/2T$$

Linear Transversal Filter

- As ISI is limited to a finite number of samples, the channel equalizer can be approximated by a finite impulse response (FIR) filter or a transversal filter

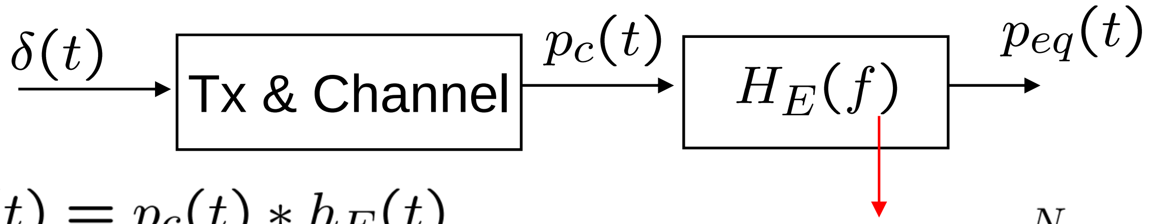


(2N+1)-tap FIR equalizer

- $\{c_n\}$ are the adjustable 2N+1 equalizer coefficients
- N is chosen sufficiently large so that the equalizer spans the length of ISI

Zero-Forcing Equalizer

- Let $p_c(t)$ denote the received pulse from a channel to be equalized



$$p_{eq}(t) = p_c(t) * h_E(t)$$
$$= \sum_{n=-N}^{N} c_n p_c(t - nT)$$

$$h_E(t) = \sum_{n=-N}^{N} c_n \delta(t - nT)$$

- At sampling time t = mT

$$p_{eq}(mT) = \sum_{n=-N}^{N} c_n p_c[(m-n)t] = \begin{cases} 1, & m = 0 \\ 0, & m = \pm 1, \ldots, \pm N \end{cases}$$

To suppress 2N adjacent interference terms

- Rearrange to matrix form

$$\mathbf{p}_{eq} = \mathbf{P}_c \cdot \mathbf{c}$$

where

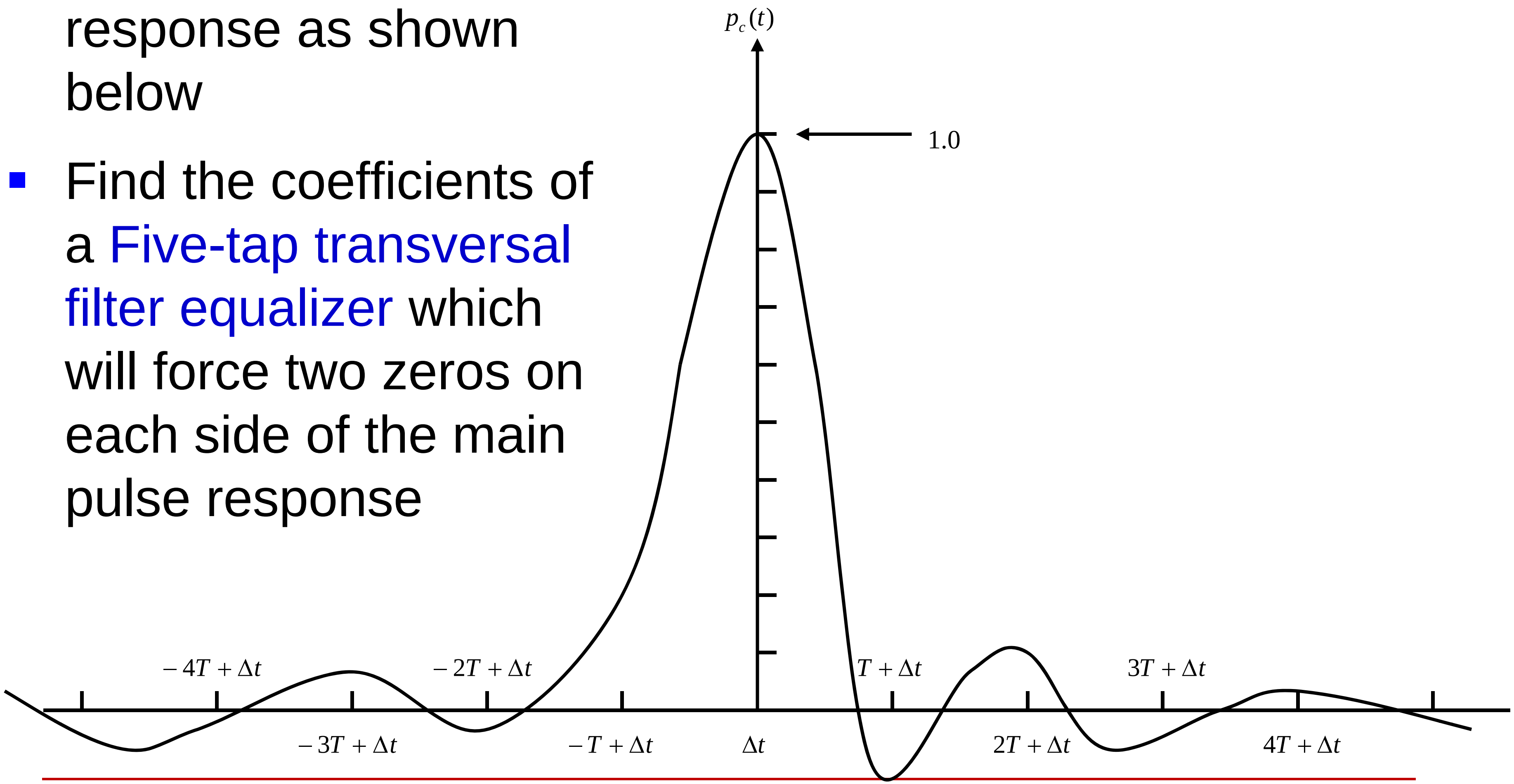
(2N+1) x (2N+1) channel response matrix

$$\mathbf{p}_{eq} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} c_{-N} \\ c_{-N+1} \\ \vdots \\ c_{-1} \\ c_0 \\ c_1 \\ \vdots \\ c_N \end{bmatrix} \quad \mathbf{P}_c = \begin{bmatrix} p_c(0) & p_c(-1) & \cdots & p_c(-2N) \\ p_c(1) & p_c(0) & \cdots & p_c(-2N+1) \\ \vdots & \vdots & \ddots & \vdots \\ p_c(2N) & p_c(2N-1) & \cdots & p_c(0) \end{bmatrix}$$

- Thus, given $p_c(t)$, one can determine the (2N+1) unknown coefficients $\{c_{-N}, \ldots, c_0, \ldots, c_N\}$
- We have exactly N zeros on both sides of main pulse response

Example

- Consider the channel response as shown below
- Find the coefficients of a Five-tap transversal filter equalizer which will force two zeros on each side of the main pulse response



Solution

- By inspection

$$p_c(-4) = -0.02 \qquad p_c(0) = 1$$
$$p_c(-3) = 0.05 \qquad p_c(1) = -0.1$$
$$p_c(-2) = -0.1 \qquad p_c(2) = 0.1$$
$$p_c(-1) = 0.2 \qquad p_c(3) = -0.05$$
$$p_c(4) = 0.02$$

- The channel response matrix is

$$[P_c] = \begin{bmatrix} 1.0 & 0.2 & -0.1 & 0.05 & -0.02 \\ -0.1 & 1.0 & 0.2 & -0.1 & 0.05 \\ 0.1 & -0.1 & 1.0 & 0.2 & -0.1 \\ -0.05 & 0.1 & -0.1 & 1.0 & 0.2 \\ 0.02 & -0.05 & 0.1 & -0.1 & 1.0 \end{bmatrix}$$

- The inverse of this matrix, by numerical methods, is found to be

$$[P_c]^{-1} = \begin{bmatrix} 0.966 & -0.170 & 0.117 & -0.083 & 0.056 \\ 0.118 & 0.945 & -0.158 & 0.112 & -0.083 \\ -0.091 & 0.133 & 0.937 & -0.158 & 0.117 \\ 0.028 & -0.095 & 0.133 & 0.945 & -0.170 \\ -0.002 & 0.028 & -0.091 & 0.118 & 0.966 \end{bmatrix}$$

- The coefficient vector is the center column of $[P_c]^{-1}$. Therefore,

c_1=0.117, c_{-1}=-0.158, c_0 = 0.937, c_1 = 0.133, c_2 = -0.091

- The sample values of the equalized pulse response

$$p_{eq}(m) = \sum_{n=-2}^{2} c_n p_c(m-n)$$

- It can be verified that

$$p_{eq}(0) = 1.0 \qquad p_{eq}(m) = 0,\ m = \pm 1, \pm 2$$

- Note that values of $p_{eq}(n)$ for $n < -2$ or $n > 2$ are not zero. For example:

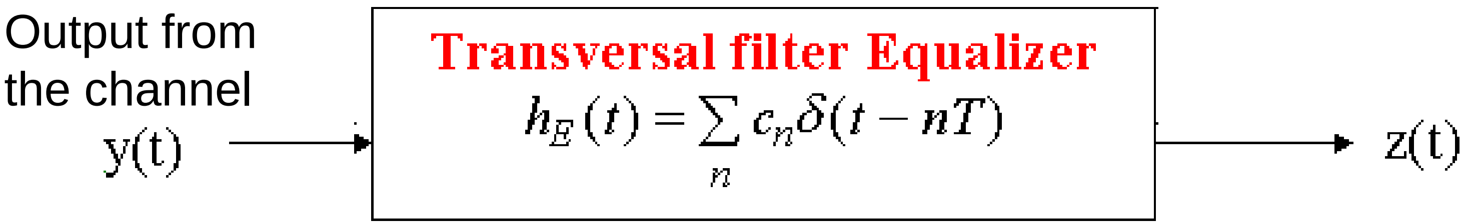
$$\begin{aligned} p_{eq}(3) &= (0.117)(0.005) + (-0.158)(0.02) + (0.937)(-0.05) \\ &\quad + (0.133)(0.1) + (-0.091)(-0.1) \\ &= -0.027 \end{aligned}$$

$$\begin{aligned} p_{eq}(-3) &= (0.117)(0.2) + (-0.158)(-0.1) + (0.937)(-0.05) \\ &\quad + (0.133)(0.1) + (-0.091)(-0.01) \\ &= 0.082 \end{aligned}$$

Minimum Mean-Square Error Equalizer

- Drawback of ZF equalizer
  - Ignores the additive noise, may result in significant noise enhancement in certain frequency range
- Alternatively,
  - Relax zero ISI condition
  - Select equalizer characteristics such that the combined power in the residual ISI and additive noise at the output of the equalizer is minimized
  - A channel equalizer that is optimized based on the minimum mean-square error (MMSE) criterion is called **MMSE equalizer**

MMSE Criterion



$$y(t) = \sum_{n=-\infty}^{\infty} A_n g_c(t - nT) + n(t)$$

$$z(t) = \sum_{n=-N}^{N} c_n y(t - nT)$$

The output is sampled at t = mT:

$$z(mT) = \sum_{n=-N}^{N} c_n y[(m-n)T]$$

Let A_m = desired equalizer output

$$\|MSE = E\left[\left(z(mT) - A_m\right)^2\right] = \text{Minimum}$$

$$MSE = E\left[\left(\sum_{n=-\infty}^{\infty} c_n y[(m-n)T] - A_m\right)^2\right]$$

$$= \sum_{n=-N}^{N}\sum_{k=-N}^{N} c_n c_k R_Y(n-k) - 2\sum_{k=-N}^{N} c_k R_{AY}(k) + E(A_m^2)$$

where

$$\begin{cases} R_Y(n-k) = E[y(mT-nT)y(mT-kT)] \\ R_{AY}(k) = E[y(mT-kT)A_m] \end{cases}$$

Expectation is taken over random sequence Am and the additive noise

- MMSE solution is obtained by $\dfrac{\partial MSE}{\partial c_n} = 0$

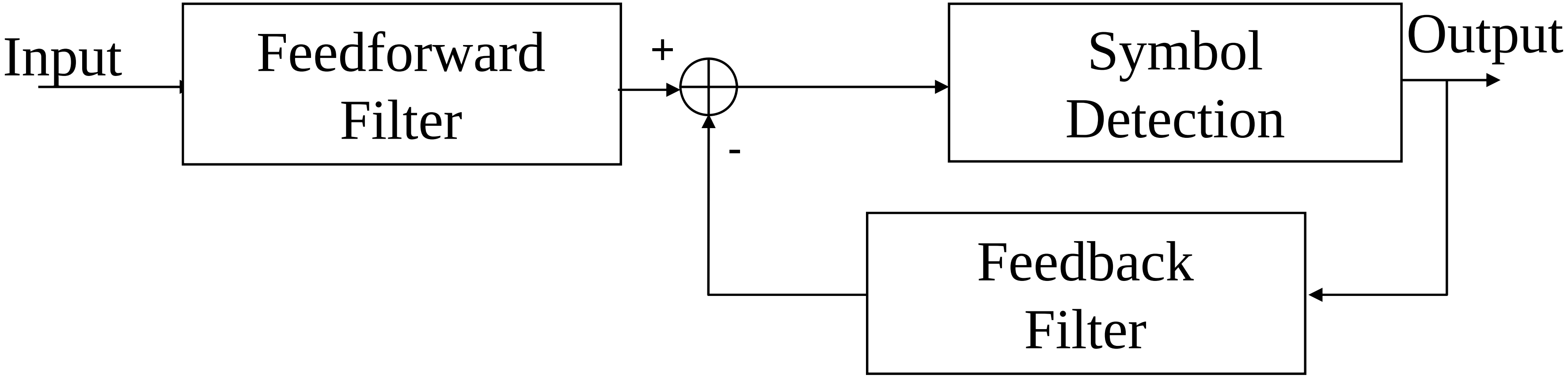
$$\Rightarrow \sum_{n=-N}^{N} c_n R_Y(n-k) = R_{AY}(k), \quad \text{for } k = 0, \pm 1, \ldots, \pm N.$$

MMSE Equalizer vs. ZF Equalizer

- Both can be obtained by solving similar equations
- ZF equalizer does not take into consideration effects of noise
- MMSE equalizer designed so that mean-square error (consisting of ISI terms and noise at the equalizer output) is minimized
- Both equalizers are known as **linear equalizers**

Decision Feedback Equalizer (DFE)

- DFE is a nonlinear equalizer which attempts to subtract from the current symbol to be detected the ISI created by previously detected symbols



Example of Channels with ISI

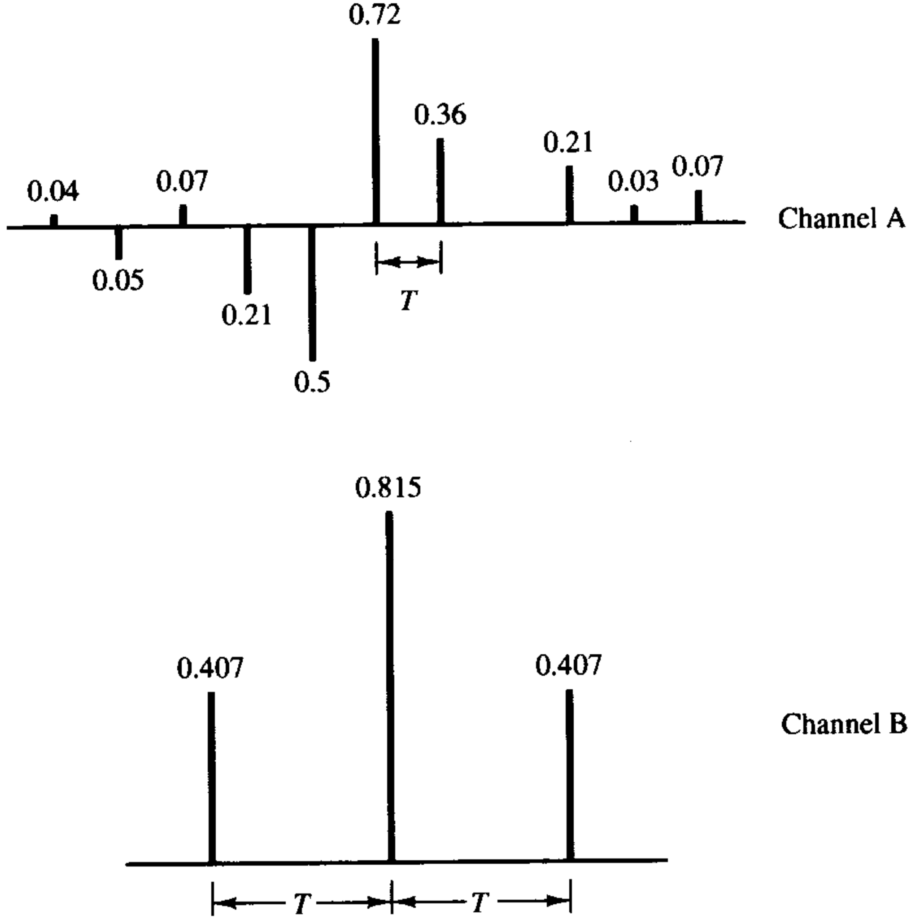


FIGURE 8.26. Two channels with ISI.

Frequency Response

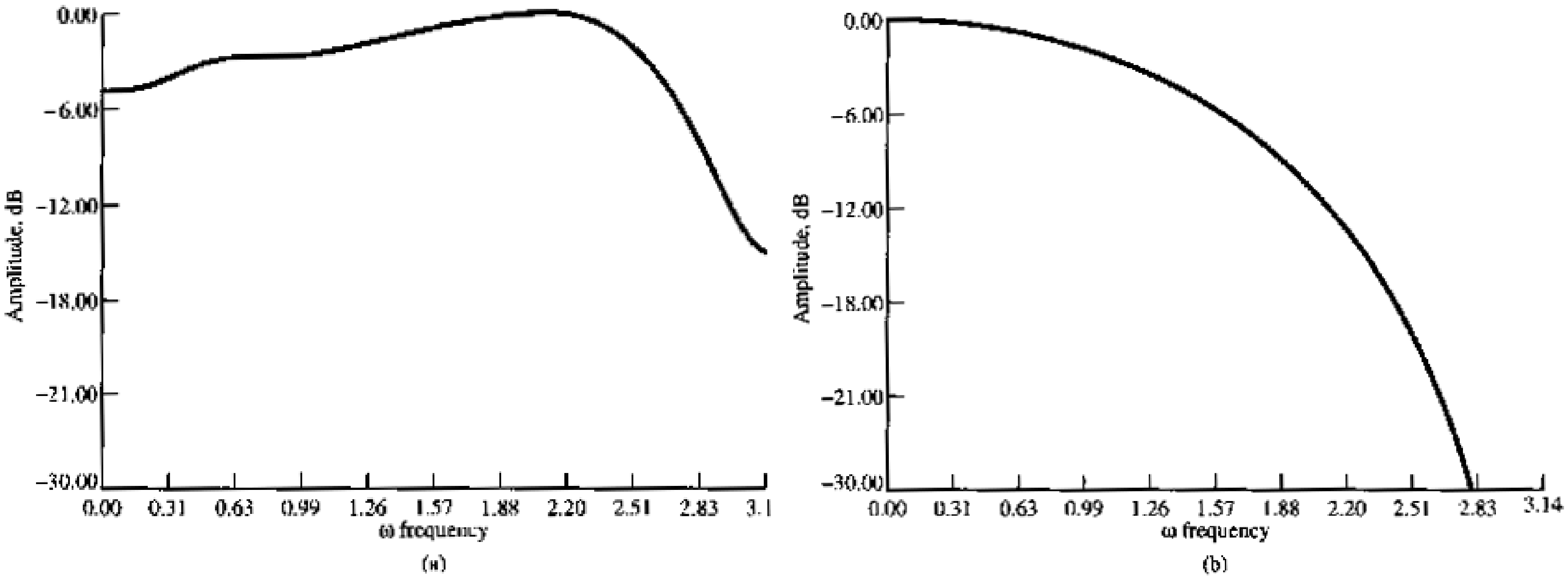


FIGURE 8.27. Amplitude spectra for (a) channel A shown in Figure 8.26(a) and (b) channel B shown in Figure 8.26(b).

Channel B will tend to significantly enhance the noise when a linear equalizer is used (since this equalizer will have to introduce a large gain to compensate channel null).

Performance of MMSE Equalizer

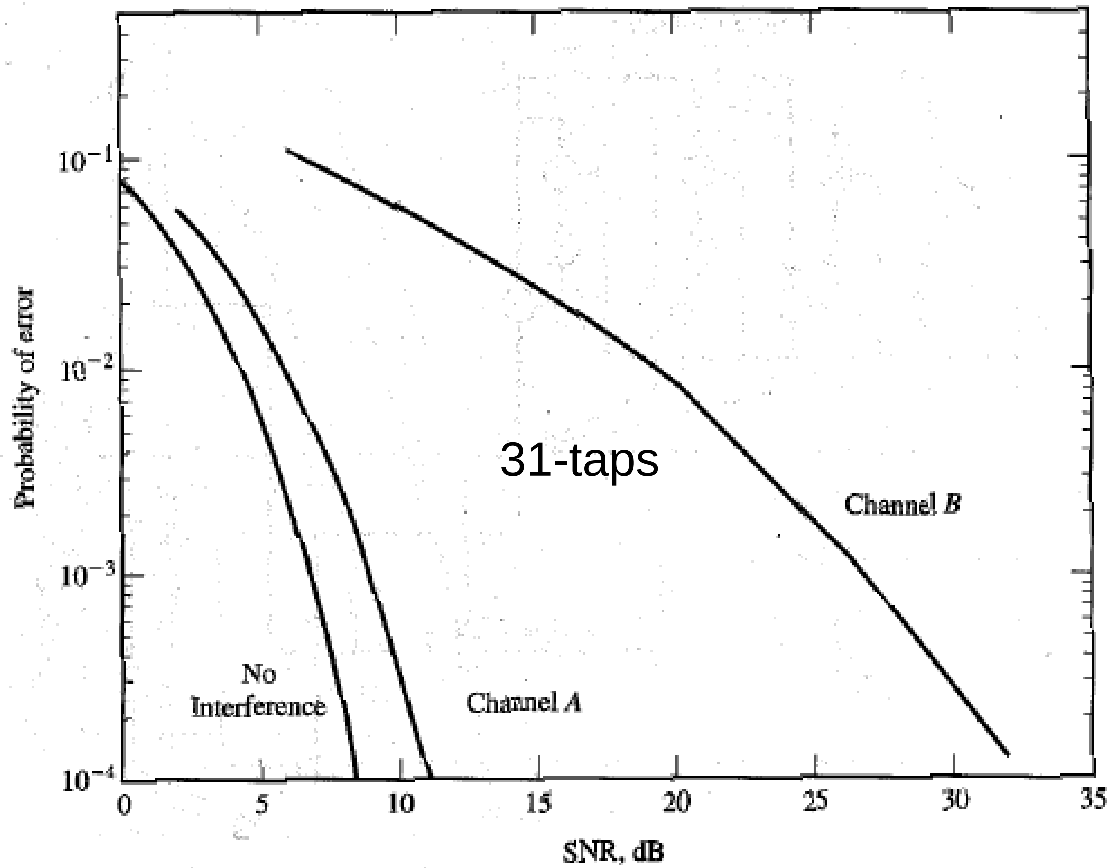
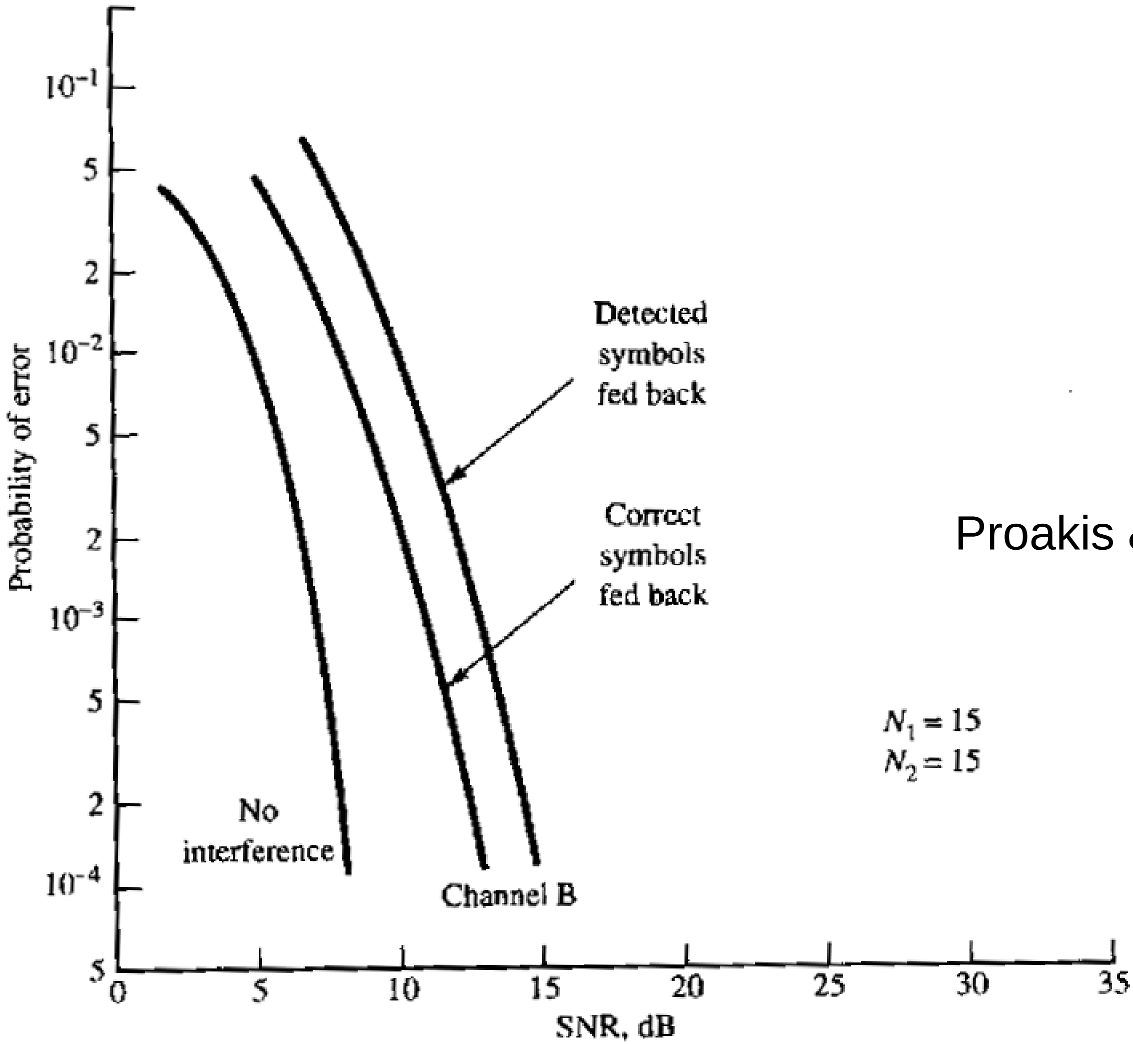


Figure 8.44 Error-rate performance of linear MSE equalizer.

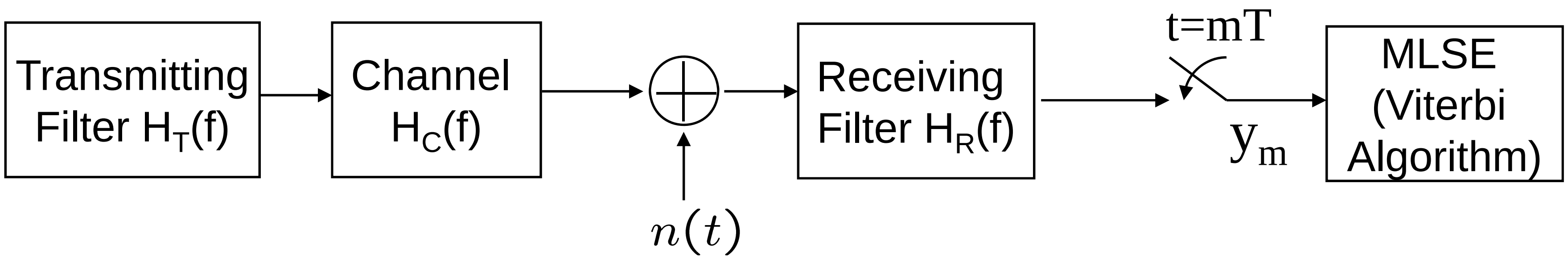
Proakis & Salehi, 2nd

Performance of DFE



Proakis & Salehi, 2^{nd}

Maximum Likelihood Sequence Estimation (MLSE)



- Let the transmitting filter have a square root raised cosine frequency response

$$|H_T(f)| = \begin{cases} \sqrt{P(f)} & |f| \leq W \\ 0 & |f| > W \end{cases}$$

- The receiving filter is matched to the transmitter filter with

$$|H_R(f)| = \begin{cases} \sqrt{P(f)} & |f| \leq W \\ 0 & |f| > W \end{cases}$$

- The sampled output from receiving filter is

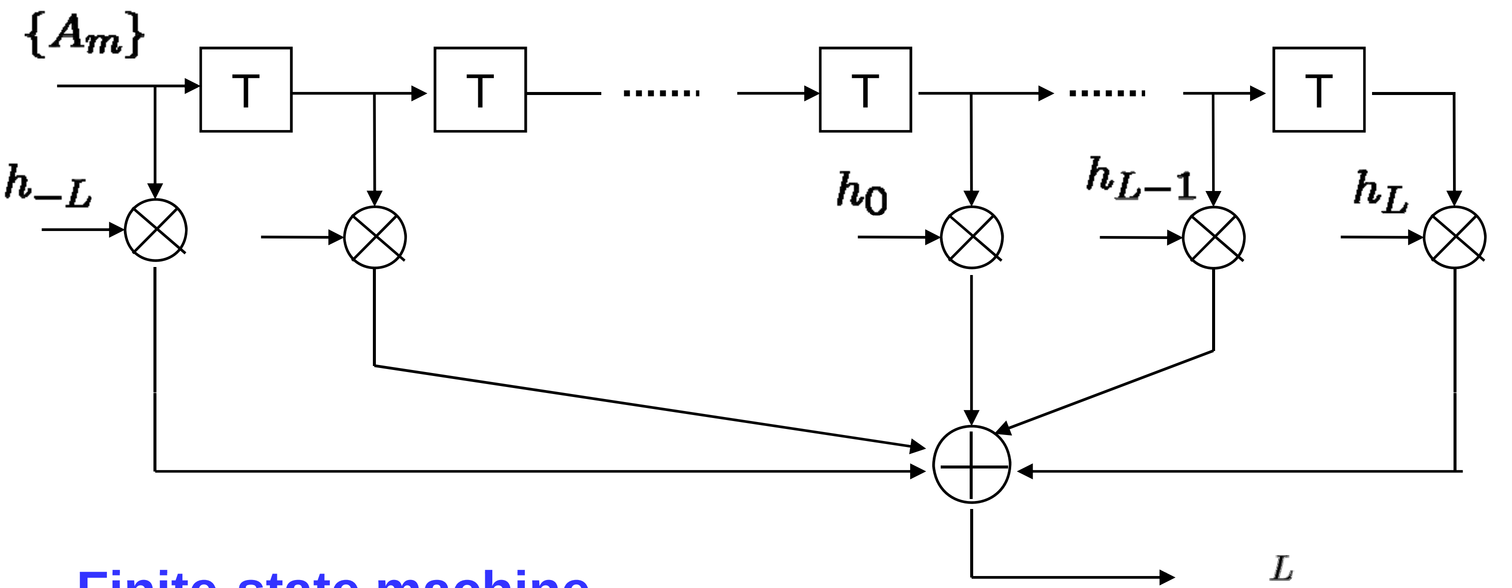
$$y_m = h_0 A_m + \sum_{\substack{n=-\infty \\ n \neq m}}^{\infty} h_{m-n} A_n + v_m$$

MLSE

- Assume ISI affects finite number of symbols, with

$$h_n = 0 \text{ for } |n| > L$$

- Then, the channel is equivalent to a FIR discrete-time filter



Finite-state machine

$$\text{Output} = \sum_{k=-L}^{L} h_k A_{m-k}$$

Performance of MLSE

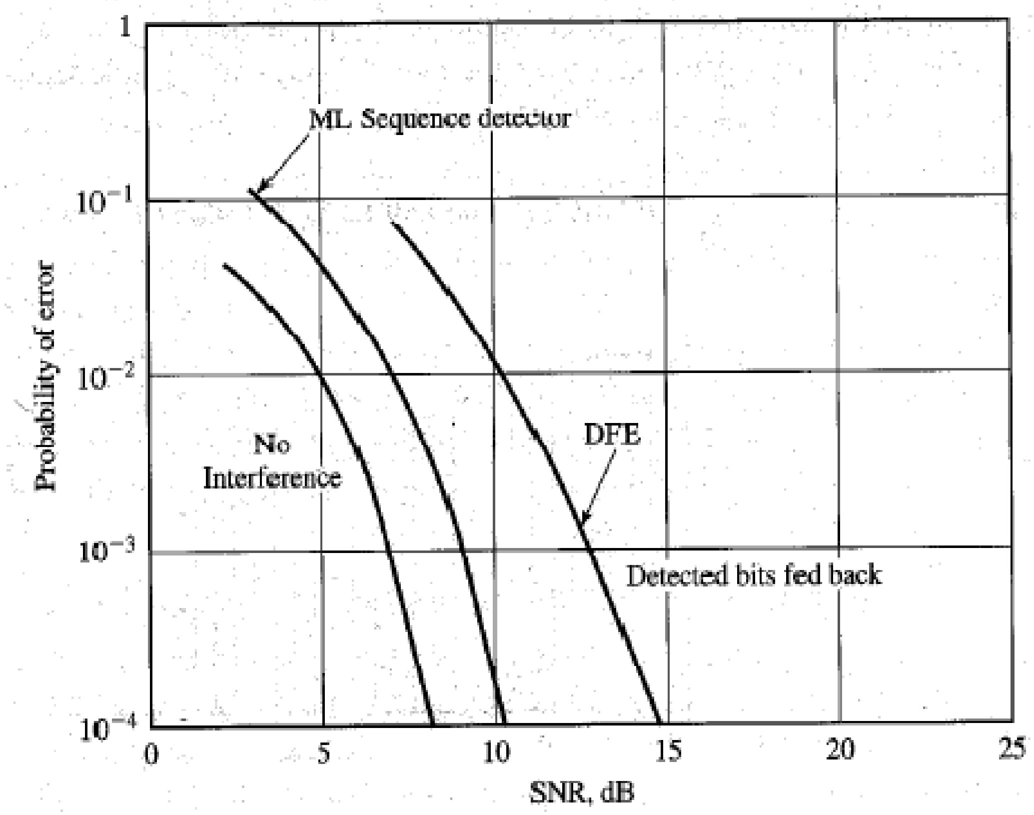
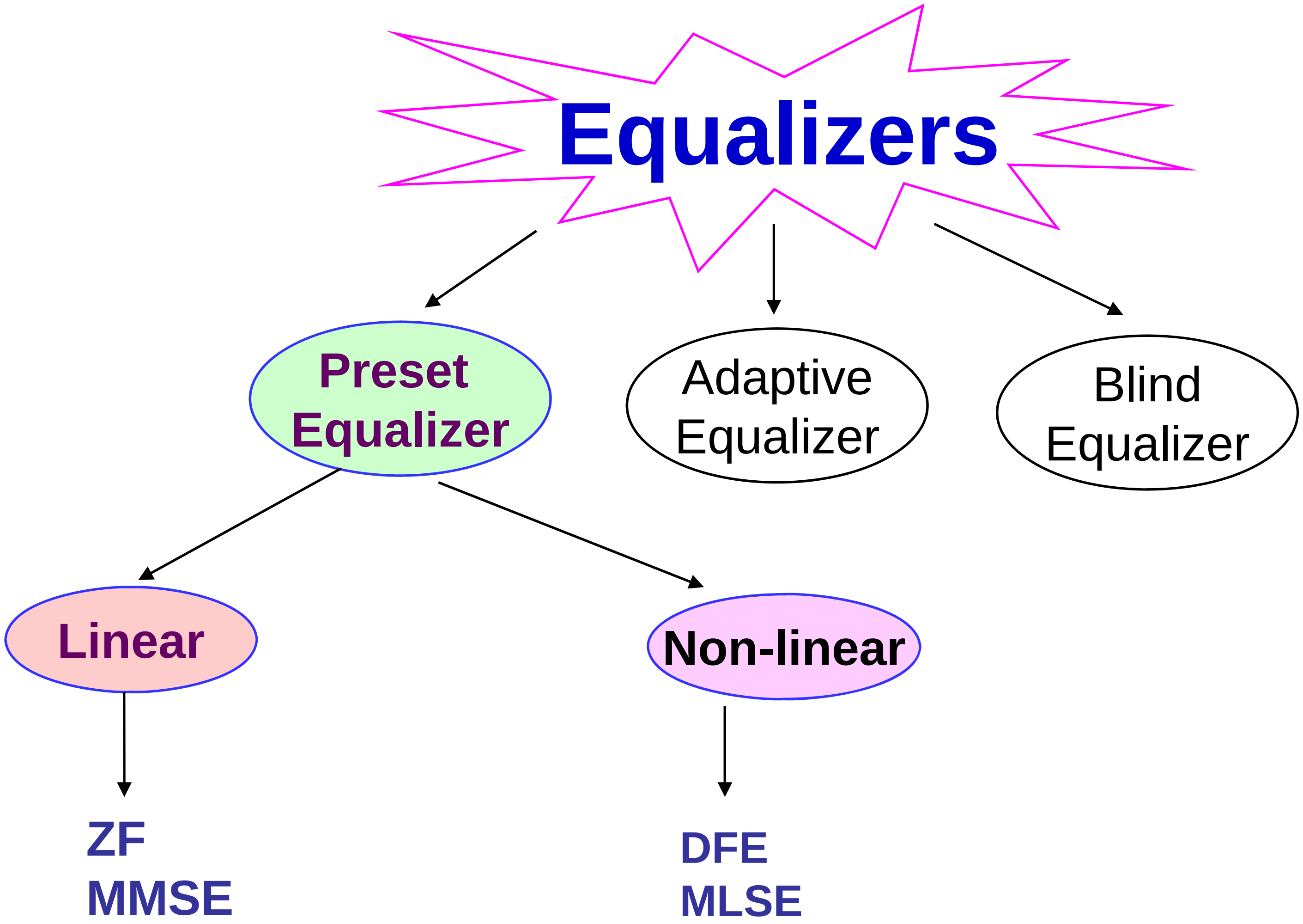


Figure 8.48 Performance of Viterbi detector and DFE for channel B.

Proakis & Salehi, 2nd


Equalizers
Preset Equalizer
Adaptive Equalizer
Blind Equalizer
Linear
Non-linear
ZF
MMSE
DFE
MLSE

Homework 3

- Textbook Chapter 7: **7.4, 7.18**
- Textbook Chapter 9: **9.4, 9.5, 9.12, 9.24**
- Due: **In-class submission on November 7th (Monday)**

Schedule -1

Week 1	Ch01:Introduction
Week 2 Week 3	Ch02:Signal, random process, and spectra
Week 4 Week 5	Ch03:Analog modulation
Week 6	Ch04: Analog to Digital Conversion
Week 7 Week 8	Ch05: Digital transmission through baseband channels

Schedule -2

Week 9	Ch06: Signal space presentation
Week 10	Ch07: Optimal receivers
Week 11	**Tutorial and Mid-term Test**
Week 12	Ch08: Digital modulation techniques
Week 13	
Week 14	Ch09: Synchronization
Week 15	Ch10: Information theory
Week 16	Ch11: Channel Coding