Principles and Experiments of Communications

Weiyao Lin

Shanghai Jiao Tong University

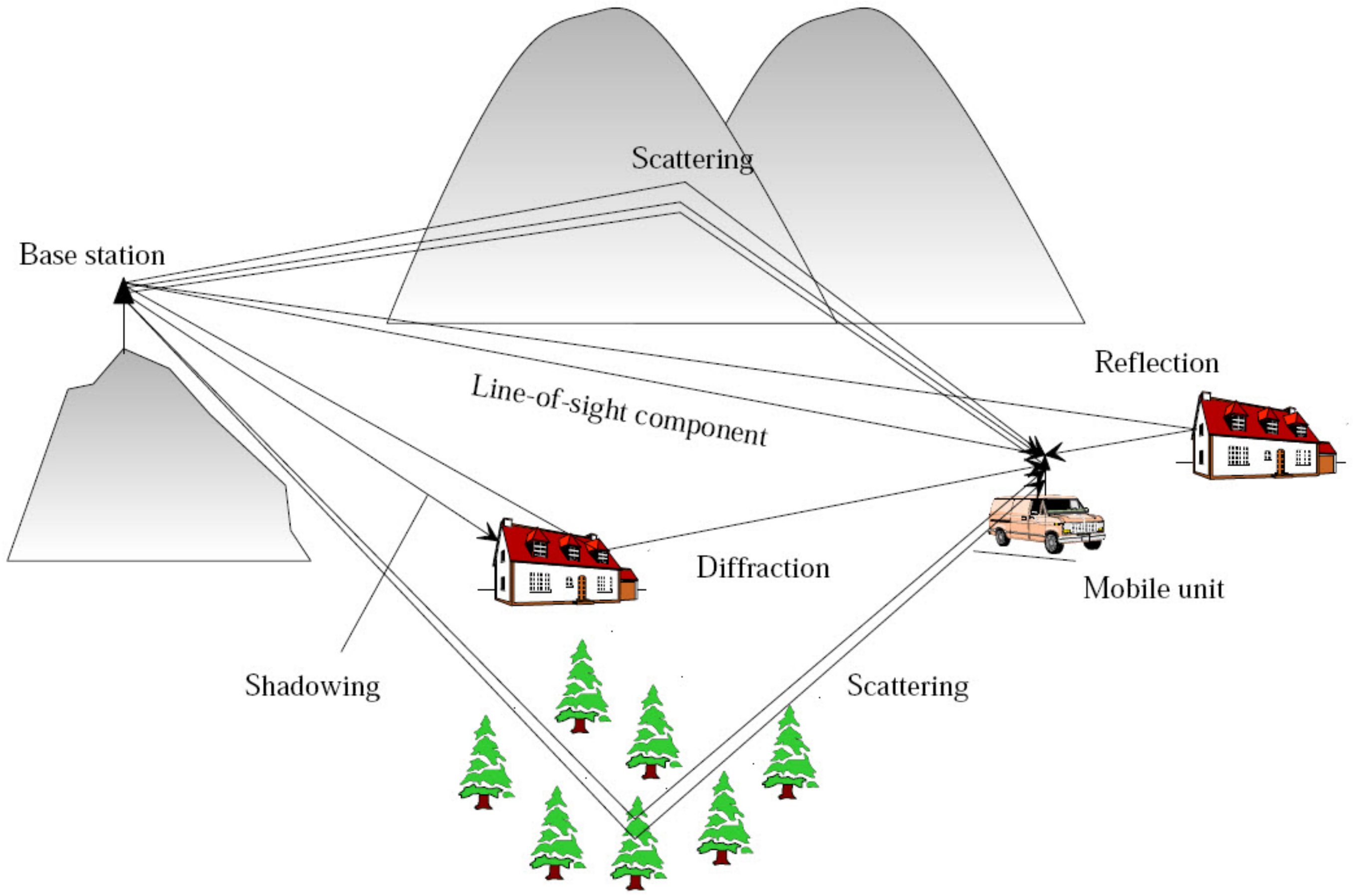
Lecture 12: Multicarrier Modulation and OFDM

Selected from Chapter 11 of *Fundamentals of Communications Systems*, 2nd edition, Pearson 2014, by Proakis & Salehi

Topics to be Covered

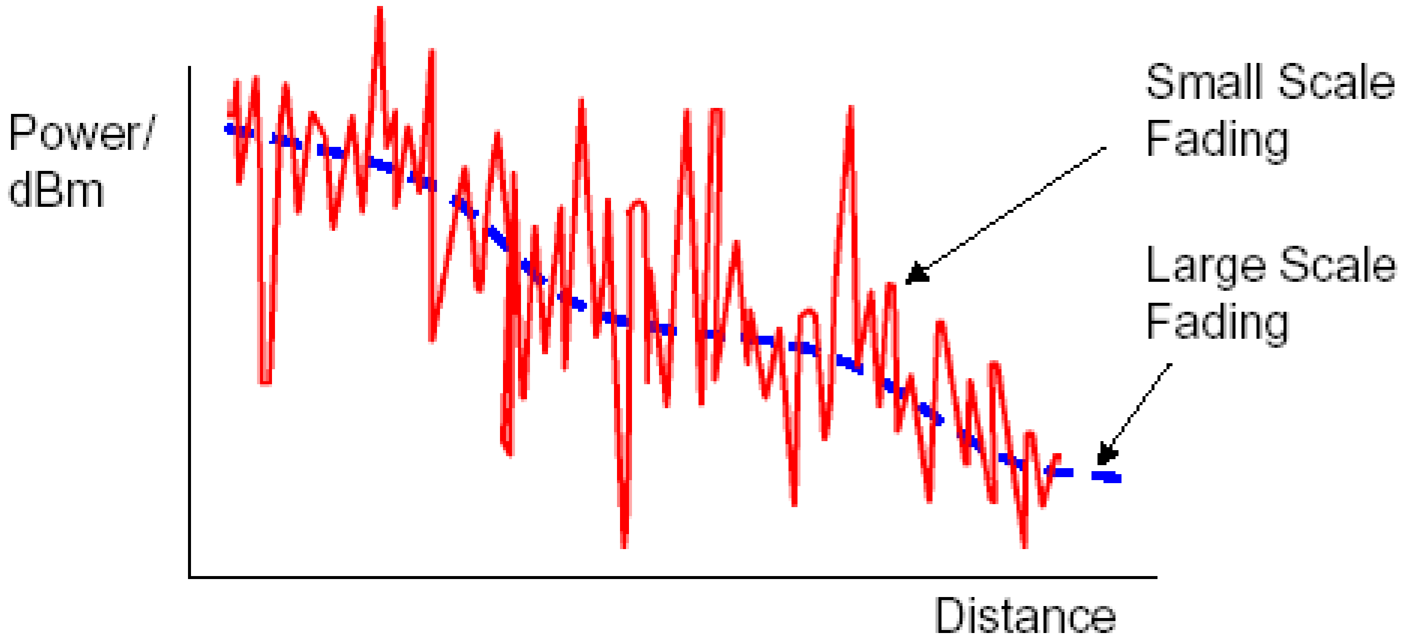
- Multipath fading channels
- Basics of multicarrier modulation
- OFDM modulation and demodulation
- Spectrum and PAPR of OFDM
- Applications of OFDM

Typical Mobile Wireless Channels



- A distinct characteristic of the mobile wireless channel is the variations of the channel strength over time and over frequency.

Large-Scale and Small-Scale Fading



- Large-scale fading: due to path loss of signal as a function of distance and shadowing
- Small-scale fading: due to the multiple paths between transmitter and receiver.

Large-Scale Fading

- The average power received by a receiver which is separated from a transmitter by distance *d:*

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^m L} \propto \left(\frac{1}{d}\right)^m \quad (m = 2 \sim 4)$$

where P_t is the transmitted power

G is the antenna gain

λ is the wavelength

L is the system loss factor

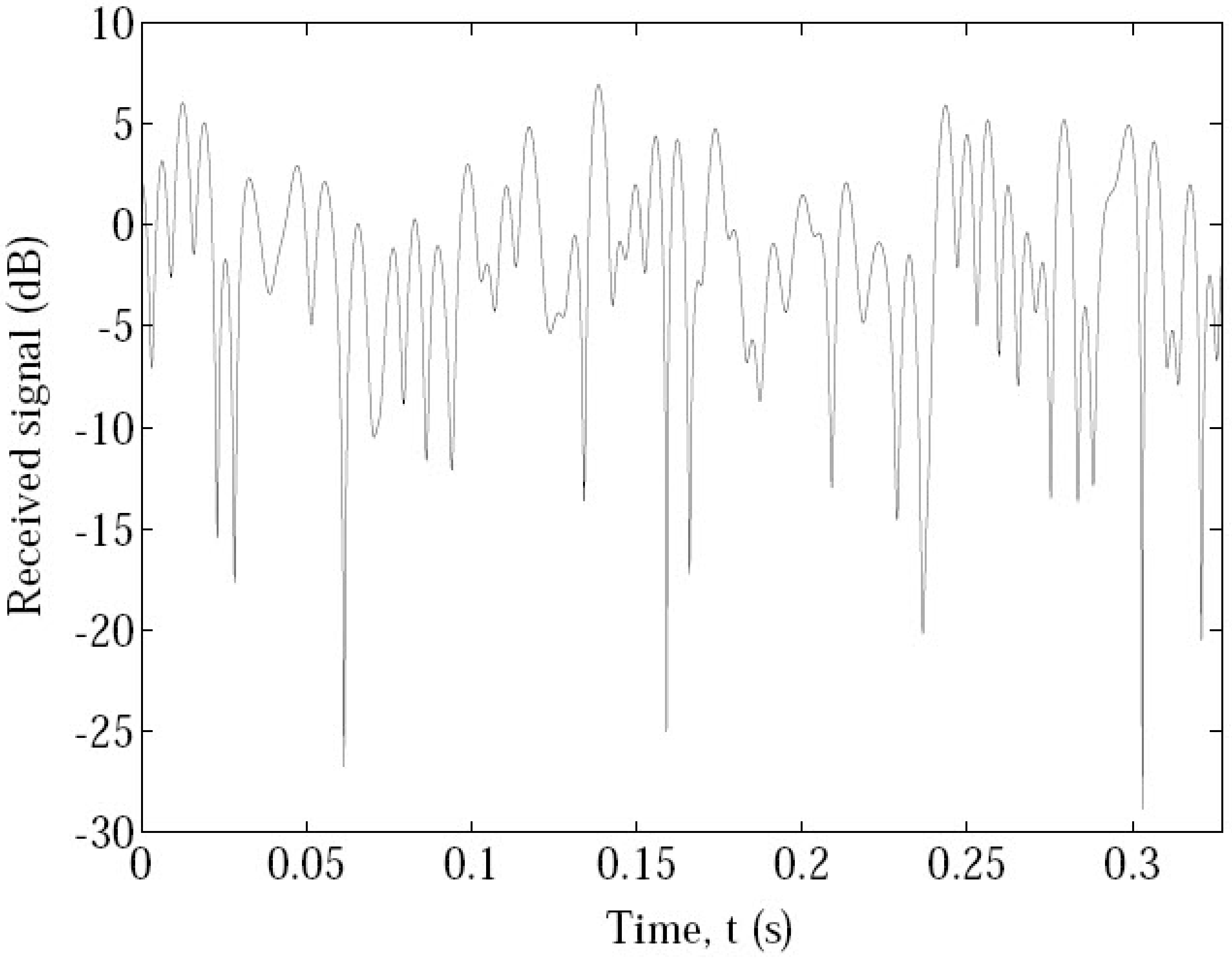
d is the transmit-receive distance

m is the path-loss exponent. In free space, $m = 2$

Small-Scale Fading

- Multipath in the wireless channel creates small-scale fading effects.
- The most important effects:
  - Rapid changes in signal strength;
  - Varying Doppler shifts;
  - Time dispersion.

Typical Behavior of Received Signals



Multipath Channels

- The transmission of an unmodulated carrier

$$s(t) = A\cos 2\pi f_c t$$

- The received signal without noise

$$r(t) = A\sum_N \alpha_n(t)\cos\left[2\pi f_c\left(t-\tau_n(t)\right)\right]$$

$$= A\,\mathrm{Re}\left[\sum_N \alpha_n(t)e^{-j2\pi f_c\tau_n(t)}e^{j2\pi f_c t}\right]$$

$$= \mathrm{Re}\left[c(t)e^{j2\pi f_c t}\right]$$

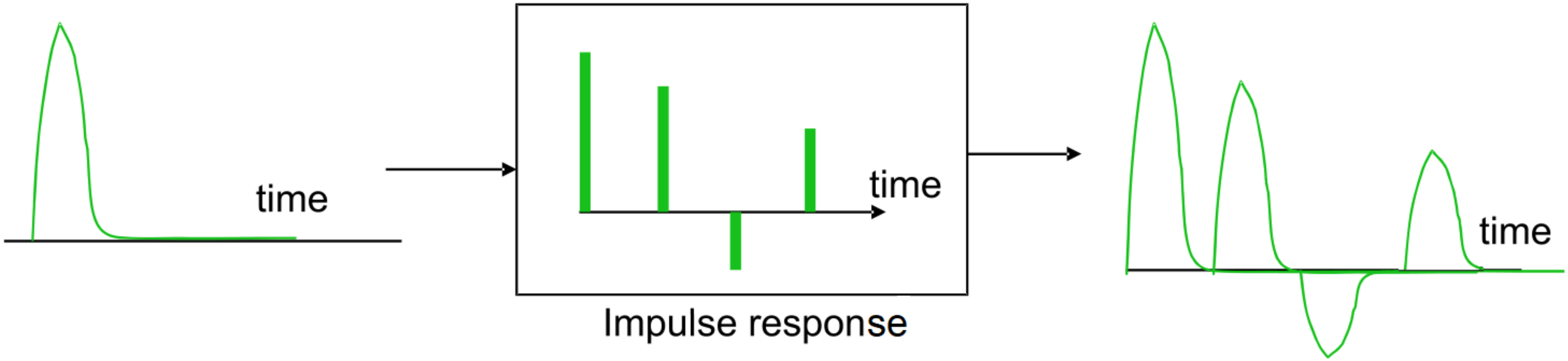
where $\alpha_n(t)$ is the time-variant attenuation factor

$\tau_n(t)$ is the corresponding propagation delay

N is the total number of paths

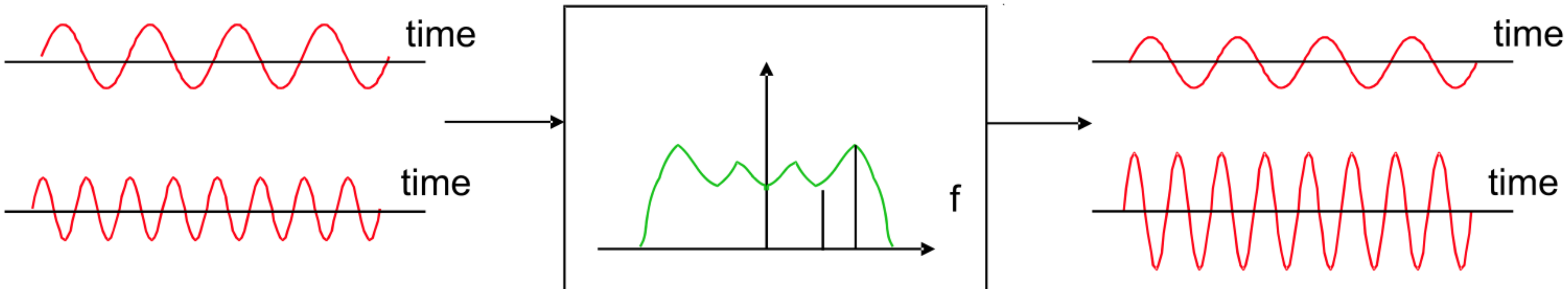
Multipath Channels

- Time domain: impulse response



Impulse response

- Frequency domain: frequency response



Sinusoidal signal as input

Frequency response

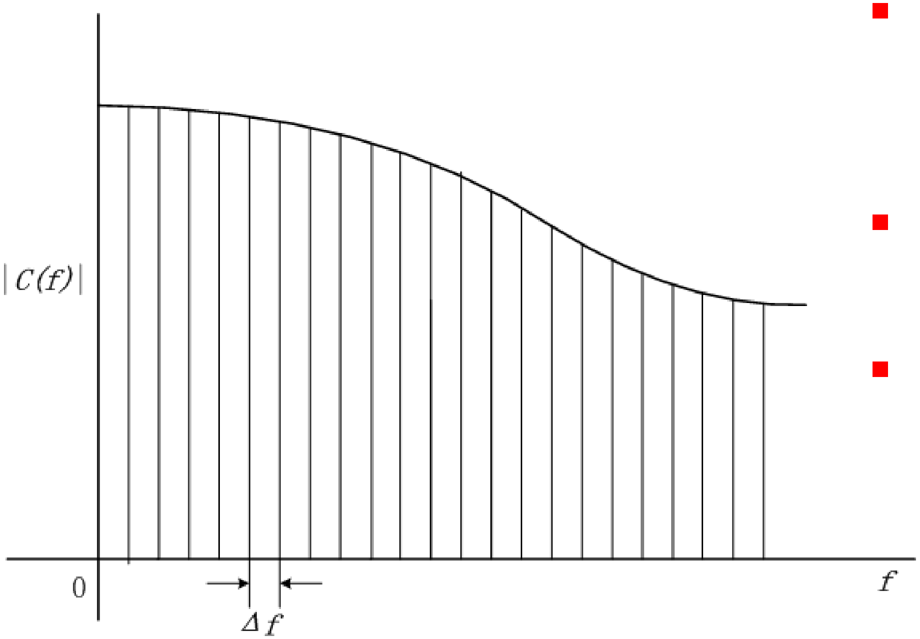
Sinusoidal signal as output

Delay Spread and Coherence Bandwidth

- Delay spread: $\tau = \max_{i,j} \left| \tau_i(t) - \tau_j(t) \right|$
  - It represents the difference in propagation time between the longest and shortest path.
- Coherence bandwidth: $B_c = \frac{1}{\tau}$
  - It can be thought of as the bandwidth during which the channel frequency response is approximately constant
- Flat fading
  - Signal bandwidth B > channel coherence bandwidth B_c
  - Symbol duration T_s > channel delay spread τ
- Frequency-selective fading
  - Signal bandwidth B < channel coherence bandwidth B_c
  - Symbol duration T_s < channel delay spread τ

Multicarrier Modulation

- Freq.-selective fading causes inter-symbol interference
- For wideband channels that provide high data rate transmission,
  - $T_s \ll \tau$ and the ISI is very severe.
- Basic idea of **multicarrier modulation**



- Subdivide the channel bandwidth into K narrowband subchannels of equal bandwidth $\triangle f = \frac{W}{K}$
- Transmit independent data streams in parallel over the K subchannels
- Consequently, the data is transmitted by frequency-division multiplexing (FDM)

Multicarrier Modulation

- With each subchannel k, we associate a subcarrier

$$x_k(t) = \cos 2\pi f_k t, \;\; k = 0, 1, \ldots, K-1$$

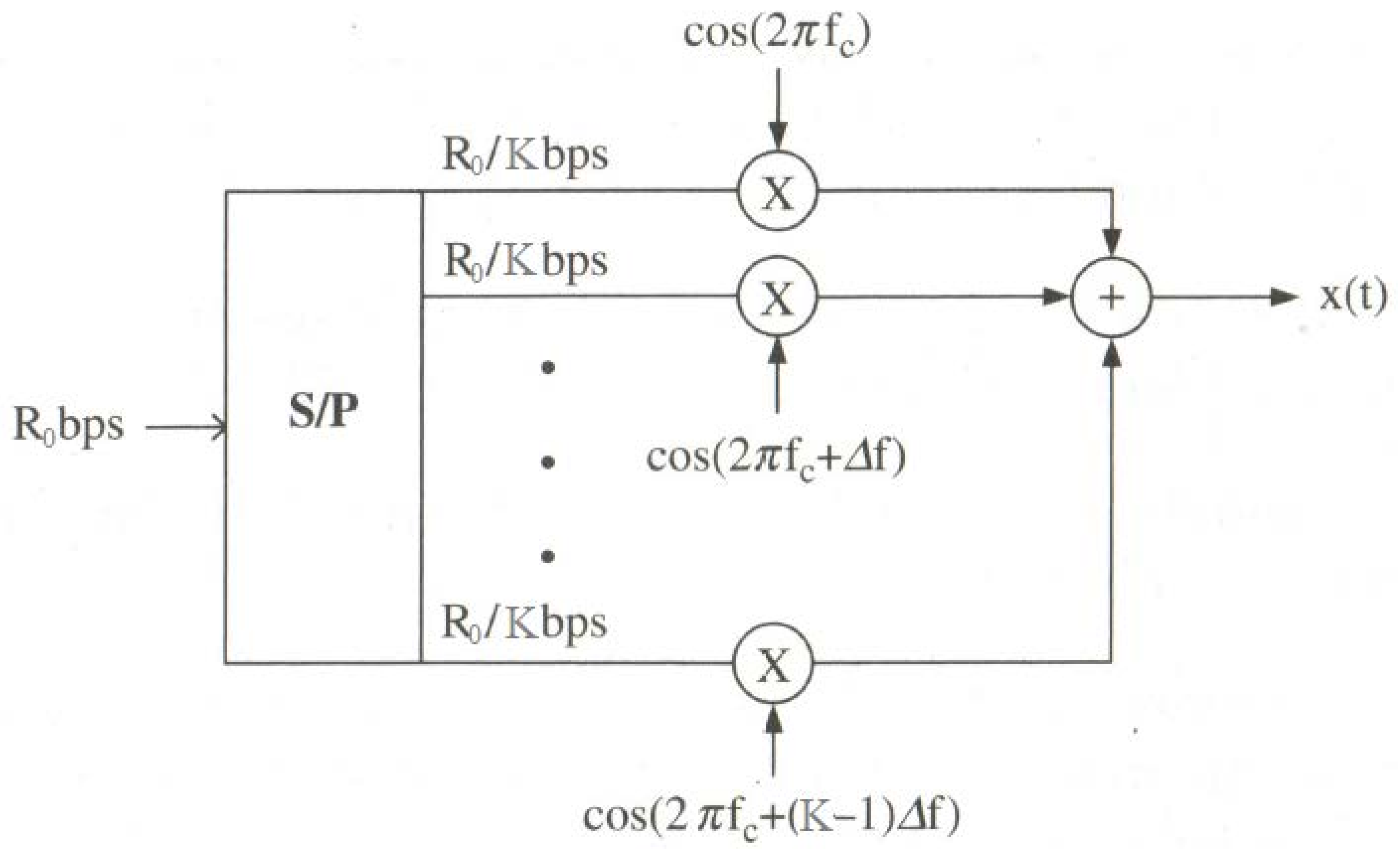
- By selecting the symbol rate $1/T$ on each subcarrier to be the separation $\triangle f$ of the adjacent subcarriers, the subcarriers are orthogonal over the symbol interval T .
- Multicarrier modulation is often referred to as OFDM if the subcarriers are orthogonal.
- With sufficiently large K , for each subcarrier:
  - In time domain, symbol time $T = KT_s \gg \tau$

    Each subcarrier is hence effectively ISI-free.
  - In frequency domain, subcarrier bandwidth $B/K \ll B_c$

    Each subcarrier experiences relatively flat fading.

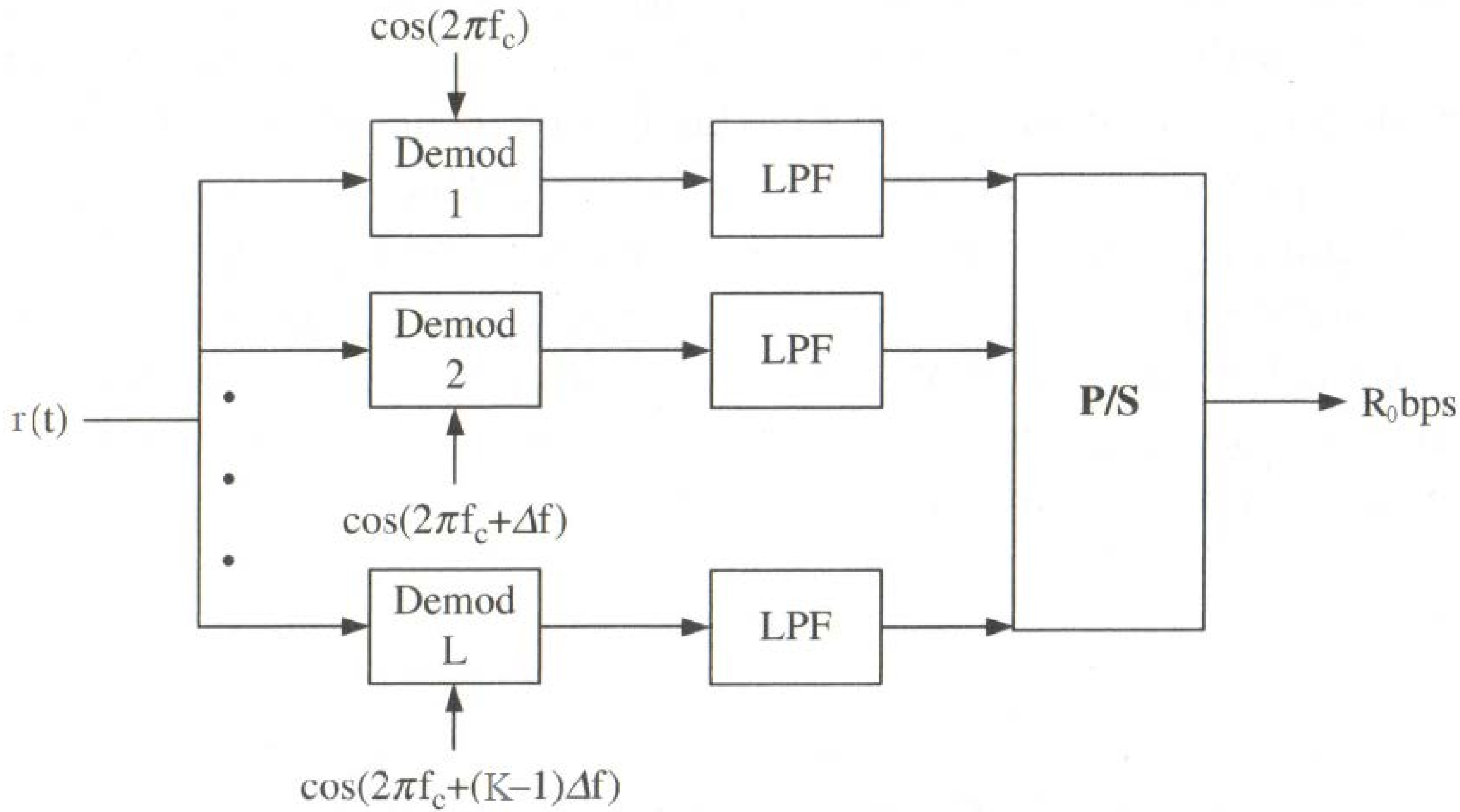
Transmitter

- Basic multicarrier transmitter



Receiver

- Basic multicarrier receiver



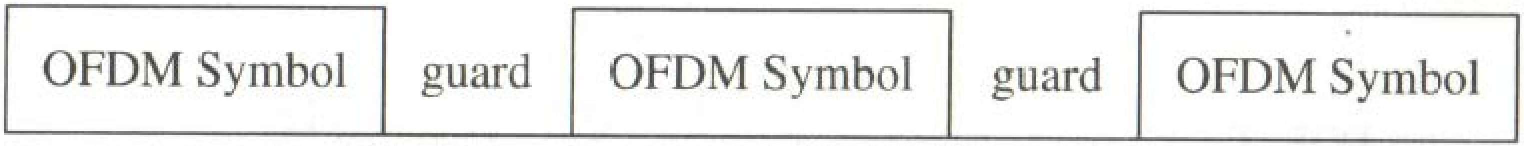
OFDM

- Shortcoming of the basic multicarrier modulation:
  - Require K independent RF units and demodulation paths.
- The process of multicarrier modulation and demodulation is equivalent to discrete Fourier transform (DFT) and its inverse
- OFDM employs an efficient computational of DFT by FFT
- FFT and IFFT algorithms:
  - The key to making OFDM realizable in practice
  - Complexity of DFT: $O(K^2)$

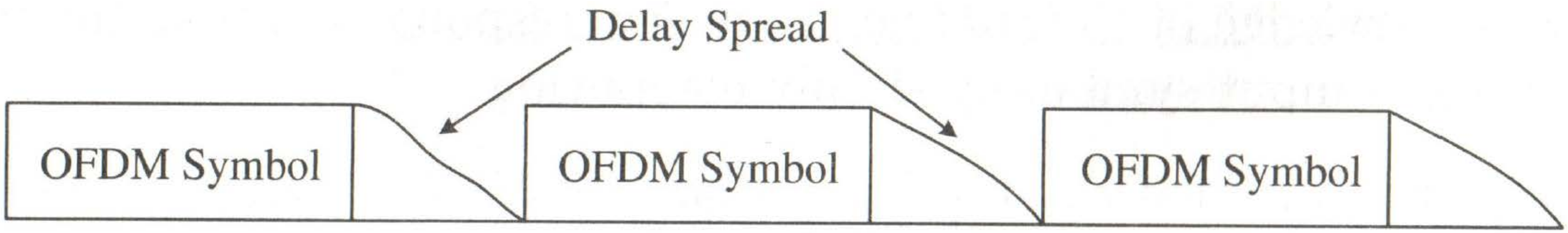
    Complexity of FFT: $O(K \log K)$
  - Allow all the subcarriers to be created in the digital domain, and thus require only a single radio to be used.

Block Transmission with Guard Intervals

- An OFDM symbol: group K data symbols into a block
- Introduce a guard time T_g between each OFDM symbol
  - In order to keep each OFDM symbol independent of the others after going through a channel.



  - If $T_g > \tau$, each OFDM symbol will only interfere with itself.



Circular Convolution and DFT

- Compute the output by circular convolution:

$$r[n] = x[n] \circledast c[n] = c[n] \circledast x[n] \triangleq \sum_{k=0}^{K-1} c[k]x[n-k]_K$$

  - $x[n]_K = x[n \bmod K]$ is a periodic version of $x[n]$ with period K

- Take the DFT:

$$\mathrm{DFT}\{r[n]\} = \mathrm{DFT}\{c[n] \circledast x[n]\}$$

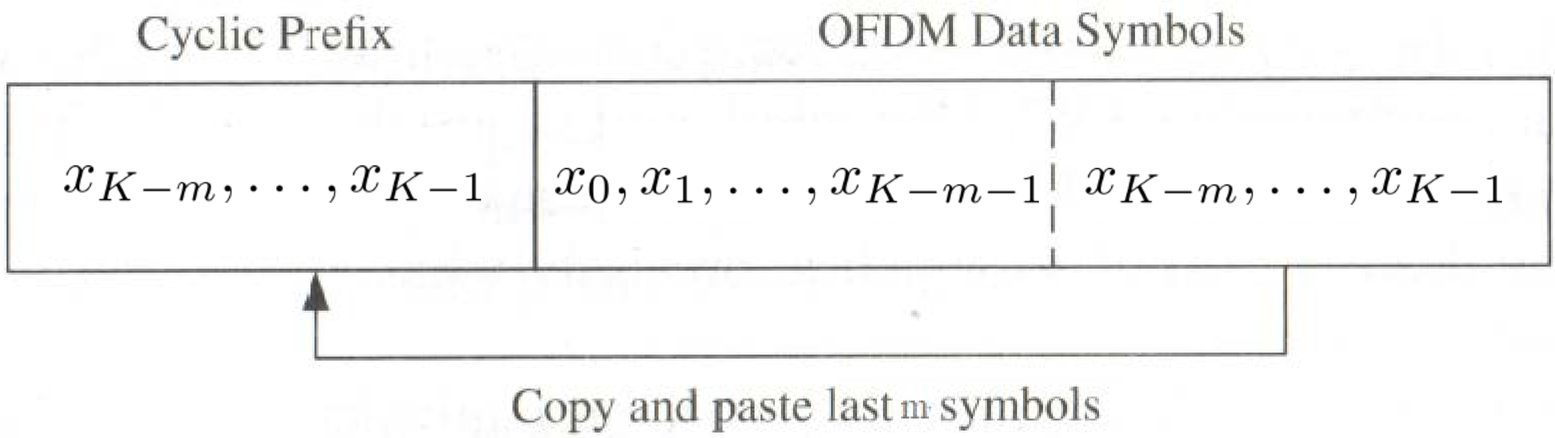
  - K point DFT and IDFT:

$$\mathrm{DFT}\{x[n]\} = X[m] \triangleq \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} x[n] e^{-j\frac{2\pi nm}{K}}$$

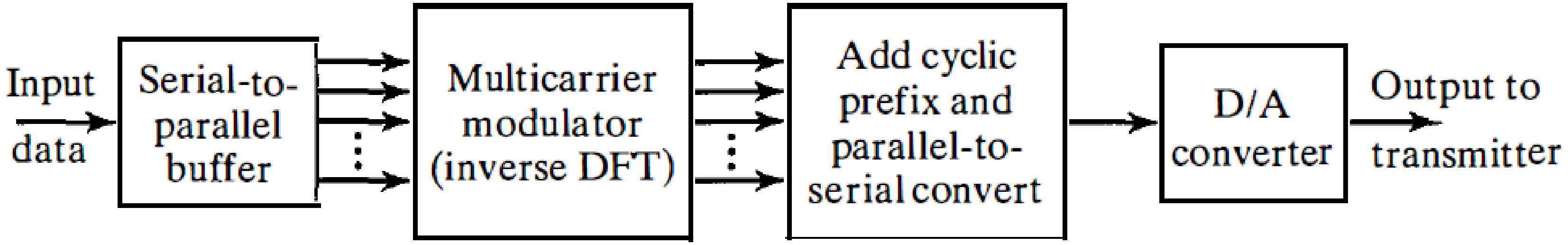
$$\mathrm{IDFT}\{X[m]\} = x[n] \triangleq \frac{1}{\sqrt{K}} \sum_{m=0}^{K-1} X[m] e^{j\frac{2\pi nm}{K}}$$

Cyclic Prefix

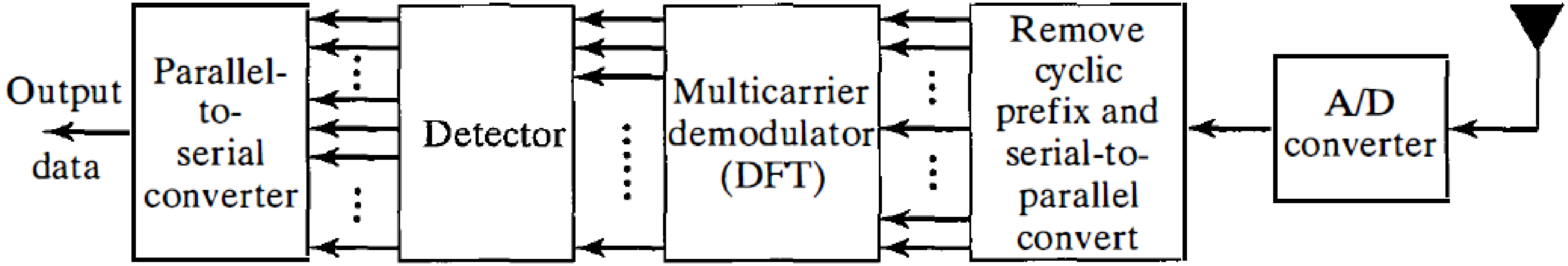
- The circular convolution can be realized by cyclic prefix.
- The maximum channel delay spread: $m+1$ samples
- Add a guard band of at least m samples between OFDM symbols.



OFDM Block Diagram



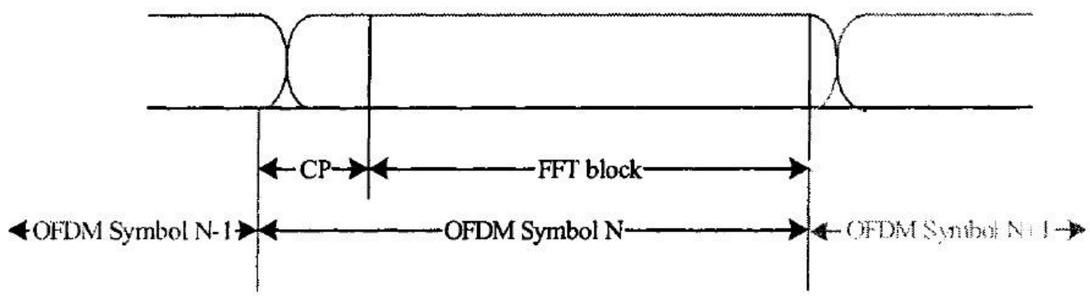
(a) Transmitter



(b) Receiver

Eliminate ISI

- An alternative approach is to append cyclic prefix to each OFDM symbol



CP: Cyclic Pre-fix

- Since the ISI in any pair of successive signal transmission block only affects the cyclic prefix, we discard the affected samples of received signal to eliminate ISI

SNR per Subchannel

- The SNR per subchannel can be defined as

$$\mathrm{SNR}_k = \frac{TP_k|C_k|^2}{\sigma_{nk}^2}$$

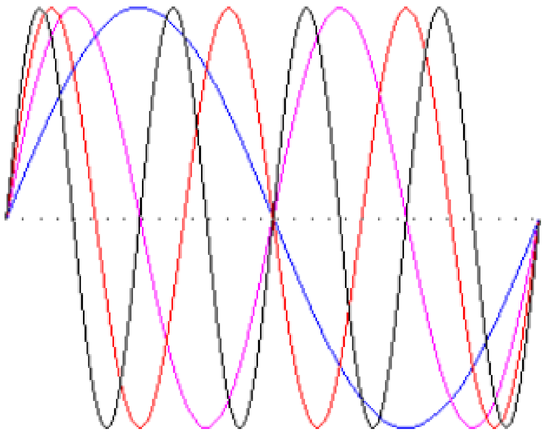
  - T : the symbol duration
  - P_k: the average transmitted power allocated to the k-th channel
  - C_k: the freq. response of the k-th subchannel
  - σ_{nk}^2 : the corresponding noise variance
- OFDM can be optimized by adaptive bit and power allocation in each subcarrier

- C. Y. Wong, R.S. Cheng, K. B. Letaif, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit and power allocation," *IEEE JSAC*, Oct. 1999.
- M. Tao, Y. C. Liang, and F. Zhang, "Resource allocation for delay differentiated traffic in multiuser OFDM systems", *IEEE TWC*, June 2008.

Spectrum of OFDM Signal

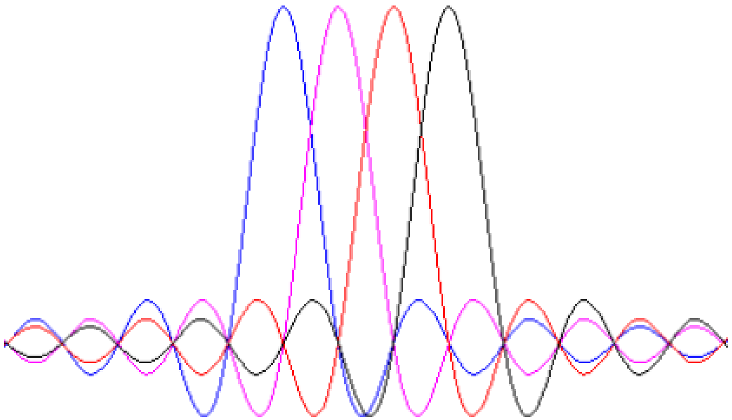
- OFDM signals are orthogonal in the time domain but have significant overlap in frequency domain.

Time domain



Example of four subcarriers within one OFDM symbol

Frequency domain



Spectra of individual subcarriers

Peak-to-Average Power Ratio (PAPR)

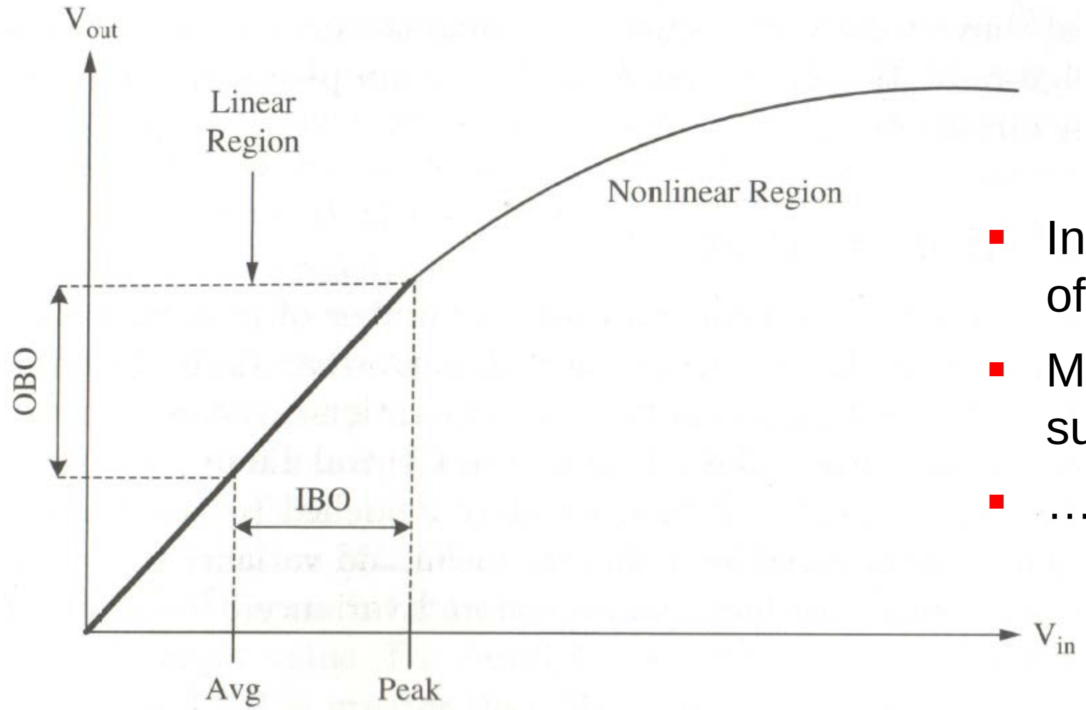
- OFDM signals have a high PAPR
  - A multicarrier signal is the sum of many narrowband signals in the time domain; thus the peak value of the signal is substantially larger than the average value.
- The PAPR of OFDM is defined for the IFFT output

$$\text{PAPR} \triangleq \frac{\max_{l \in (1, K+N_g)} |x_l|^2}{E[|x_l|^2]}$$

- High PAPR is the most important implementation challenge of OFDM
  - It reduces the efficiency and increases the cost of the RF power amplifier
  - Alternatively, the same PA can be used but the input power must be reduced
  - This is known as input backoff and results in lower average received SNR

PAPR Problem

- A typical power amplifier (PA) response
  - Operation in the linear region is required in order to avoid distortion, so the peak value must be constrained in this region.
  - The PA is underutilized by a "backoff" amount



Solutions

- Insert different phase shifts in each of the subcarriers.
- Modulate a small subset of the subcarriers with dummy symbol.
- …

Applications of OFDM

- Digital audio broadcasting (DAB)
- Digital video broadcasting (DVB)
- Digital subscriber lines (DSL)
- Wireless local area networks (LANs)
- Long-term evolution (LTE)

OFDM Parameters in LTE

TABLE 1 LTE DOWNLINK PHYSICAL LAYER PARAMETERS.

Channel Bandwidth (MHz)	1.25	2.5	5	10	15	20
Frame Duration (ms)	10					
Subframe Duration (ms)	1					
Sub-carrier Spacing (kHz) Δf	15					
Sampling Frequency (MHz)	1.92	3.84	7.68	15.36	23.04	30.72
FFT Size	128	256	512	1024	1536	2048
Occupied Sub-carriers (inc. DC sub-carrier)	76	151	301	601	901	1201
Guard Sub-carriers	52	105	211	423	635	847
Number of Resource Blocks	6	12	25	50	75	100
Occupied Channel Bandwidth (MHz)	1.140	2.265	4.515	9.015	13.515	18.015
DL Bandwidth Efficiency	77.1%	90%	90%	90%	90%	90%
OFDM Symbols/Subframe	7/6 (short/long CP)					
CP Length (Short CP) (μs)	5.2 (first symbol) / 4.69 (six following symbols)					
CP Length (Long CP) (μs)	16.67					